

THE UNITARY DUAL OF p -ADIC G_2

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Introduction. Let G_2 be a simply connected split simple group of type G_2 over a nonarchimedean field F of characteristic zero. We determine here the set \hat{G}_2 of equivalence classes of irreducible unitarizable representations of G_2 . The corresponding problem in the complex case was solved by Duflo in [D]. Recently, Vogan has solved the corresponding problem in the real case in [V1].

We expect that our results, besides being interesting by themselves, will play a role in both local and global Θ -correspondences (see [GS], [GS1], [MS], [Sa1], and [Sa2]).

Here is an outline of the paper. In the first section, we establish notation and recall basic structure results for G_2 and basic facts from representation theory of G_2 and its standard Levi factors.

A. M. Aubert in [A], J. Bernstein, and P. Schneider and U. Stuhler in [SS] considered an involution D_G on the Grothendieck group of representations of finite length of any reductive F -group G . In the second section, we recall some facts about filtration of Jacquet modules and introduce the involution D_{G_2} , in G_2 -setting.

In the third section, we consider generalized and degenerate principal series. Theorem 3.1 describes reducibility points of generalized principal series $I_\gamma(s, \delta(\chi))$ and degenerate principal series $I_\gamma(s, \chi \circ \det)$ (see Section 1 for notation), where χ is a unitary character of F^\times , and s is a real number.

In the fourth section, we obtain a classification of square integrable representations that are supported on the minimal parabolic subgroup and describe composition factors of $I_\gamma(s, \delta(\chi))$ and $I_\gamma(s, \chi \circ \det)$ (Propositions 4.1, 4.2, 4.3, 4.4, and Theorem 4.1). We use a method of Jacquet modules developed by M. Tadić. We refer to [Re] for equality of our square integrable representations, in the unramified case, to those obtained by the well-known Kazhdan-Lusztig classification and to [GS] for its importance in the local Θ -correspondence. Tempered representations supported on the minimal parabolic subgroups were classified by Keys in [Ke] and in Theorems 3.1 and 4.1 of this paper.

In the fifth section, we classify unitarizable Langlands quotients. Theorems 5.1 and 5.2 classify unitarizable nontempered Langlands quotients that are supported on the minimal parabolic subgroup. Theorem 5.3 classifies unitarizable Langlands quotients supported on maximal parabolic subgroups.