Vol. 89, No. 3 DUKE MATHEMATICAL JOURNAL

A CHARACTERIZATION OF THE PERIODIC CALLAHAN-HOFFMAN-MEEKS SURFACES IN TERMS OF THEIR SYMMETRIES

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1. Introduction. The first examples of periodic minimal surfaces with more than one end were discovered by Riemann [12] in 1867. He constructed a one-parametric family of properly embedded minimal surfaces $\{R_\lambda : \lambda > 0\}$, which are invariant under a translation T. Furthermore, the quotient R_λ/T has genus 1, two ends, and total curvature -8π. Riemann proved also that every minimal surface expressible as a union of circles in parallel planes is either a subset of some R_λ or a subset of the catenoid. Recently, López, Ritoré, and Wei [8] have characterized Riemann's examples as the only embedded minimal tori with two planar ends in $\mathbb{R}^3/T$.

These surfaces were generalized by Callahan, Hoffman, and Meeks [2] in 1989. They found examples of complete embedded minimal surfaces, with an infinite number of annular ends. This discrete family consists of a surface M_k, for each integer $k > 0$, which is invariant under a cyclic group of vertical translations $\mathscr{T}$. The quotient $M_k/\mathscr{T}$ has genus $2k+1$, two ends, and $8(k+1)$ symmetries.

The method of construction was their starting point to obtain the first known examples of embedded minimal surfaces with an infinite number of annular ends and invariant under a screw motion, in [3]. These examples, $M_k(\theta)$, were obtained by twisting the surfaces M_k by angle θ, $|\theta| < \pi/(k+1)$. For another interesting paper on the surfaces $M_k(\theta)$, see [1].

Hoffman and Wohlgemuth [7] have also obtained new examples derived from the M_k surfaces, by the insertion of *Neovius handles*.

In this paper we study in depth the surfaces M_k. First, we have proved that the period problem has only one solution (see Propositions 1 and 2 in Section 4). This fact was conjectured by Callahan, Hoffman, and Meeks in [2] and [3], and it is the first step in any classification theorem. As a consequence of this, we are able to prove the following theorem.

MAIN THEOREM. *Let $\tilde{M}$ be a properly embedded minimal surface in $\mathbb{R}^3$ satisfying the following:*

(i) *$\tilde{M}$ has an infinite number of annular ends;*

(ii) *$\tilde{M}$ is invariant under a cyclic group of translations $\mathscr{T}$;*

Received 10 June 1996. Revision received 17 September 1996.

Authors' research partially supported by Dirección General de Investigación Ciencia y Tecnologia grant number PB94-0796.