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GROUP SYSTEMS, GROUPOIDS, AND MODULI SPACES OF PARABOLIC BUNDLES

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Introduction. Moduli spaces of homomorphisms or, more generally, twisted homomorphisms from fundamental groups of surfaces to compact connected Lie groups, were connected with geometry through their identification with moduli spaces of holomorphic vector bundles (see [29]). Atiyah and Bott [2] initiated a new approach to the study of these moduli spaces by identifying them with moduli spaces of projectively flat constant central curvature connections on principal bundles over Riemann surfaces, which they analyzed by methods of gauge theory. In particular, they showed that an invariant inner product on the Lie algebra of the Lie group in question induces a natural symplectic structure on a certain smooth open stratum. Although this moduli space is a finite-dimensional object, generally a stratified space which is locally semialgebraic [19] but sometimes a manifold, its symplectic structure (on the stratum just mentioned) was obtained by applying the method of symplectic reduction to the action of an infinite-dimensional group (the group of gauge transformations) on an infinite-dimensional symplectic manifold (the space of all connections on a principal bundle).

This infinite-dimensional approach to moduli spaces has deep roots in quantum field theory [1], but it is nevertheless interesting to try to avoid the technical difficulties of infinite-dimensional analysis, by using purely finite-dimensional methods to construct the symplectic structure and to derive some of its properties. This also allows for arbitrary, not necessarily compact, Lie groups. This program has been carried forward by several authors in the past ten years, with the result being not only technical simplification, but also new insight into the geometry of the moduli spaces, especially into their singularities (see [17]–[21]). See [22] for a leisurely introduction.

To date, most of the program just described has been worked out only for compact Riemann surfaces without boundary; see, however, [16]. The purpose of this article is to extend these results and methods to the case of Riemann surfaces with a finite number of punctures or, equivalently, with a finite number of boundary components, corresponding to the study of parabolic vector bundles in the holomorphic category. Specifically, we deal with the results listed below. The references indicate sources for the closed compact case except [16] (see below):

- a description of the symplectic form in terms of the cup product on the

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