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BLOW-UP RESULTS AND LOCALIZATION OF BLOW-UP POINTS IN AN N-DIMENSIONAL SMOOTH DOMAIN

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1. Introduction. In this paper we study the behavior of positive solutions of the following problem:

$$
(1.1) \qquad \begin{cases} u_t = \Delta u & \text{in } \Omega \times (0,T) \\ \dfrac{\partial u}{\partial \eta} = f(u) & \text{in } \partial\Omega \times (0,T) \\ u(x,0) = u_0(x) & \text{in } \Omega, \end{cases}
$$

where Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$, f is C^2 increasing and positive in $\mathbb{R}_+$, and u_0 is $C^{2+\alpha}(\bar{\Omega})$, positive, and verifies $\partial u_0/\partial\eta = f(u_0)$.

Under these hypotheses, existence and uniqueness of a classical solution up to some time T were proved in [3].

For problem (1.1), it is known that for each f, the existence of global solutions only depends on the behavior of f at infinity. This problem was first studied by H. A. Levine and L. E. Payne in [2]. W. Walter [4] proved that if f is convex, a necessary and sufficient condition for global existence is $\int^{+\infty} 1/ff' = +\infty$ (for every positive initial data u_0). In 1991, J. Lopez Gomez, V. Marquez, and N. Wolanski showed that if $1/f$ is locally in L^1 at ∞ (i.e., $\int^{\infty} 1/f$ converges), then blow-up of positive solutions necessarily occurs at a finite time (at least for domains in $\mathbb{R}^2$; see [3]).

If the solutions are nonglobal (this means that the maximal interval of existence is finite, say, $(0,T)$), we have

$$
\limsup_{t \nearrow T} \|u(x,t)\|_{L^\infty(\Omega)} = +\infty
$$

and we say that the solution $u(x,t)$ blows up at time T.

In Section 2, we give a blow-up result for every f such that

$$
(1.2) \qquad \int^{+\infty} \frac{1}{f} < +\infty
$$

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