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INTEGRAL MODELS OF CERTAIN SHIMURA CURVES

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§0. Introduction. Much of the theory of the mod p reduction of modular curves is now well understood and well documented (for example, in [KM]). Although a lot is also known to the experts in the more general setting of curves attached to arbitrary indefinite quaternion algebras over $\mathbb{Q}$, less seems to have been written down. In particular, the analogues of the results of Deligne and Rapoport on the bad reduction of $X_0(p)$ and $X_1(p)$ seem not to be in the literature. Over totally real fields other than $\mathbb{Q}$, one has the article [C] of Carayol, but the case of $\mathbb{Q}$ is excluded in [C]. Theorem 4.7 and Theorem 4.10 of this paper are the analogues of the results of Deligne and Rapoport in this more general setting, and the results show a strong analogy with the modular curve case, as expected.

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§1. False elliptic curves. Fix an indefinite nonsplit quaternion algebra D over $\mathbb{Q}$, and let $d = \operatorname{disc}(D)$. By definition, d is the product of the (finitely many) primes p for which $D \otimes_{\mathbb{Q}} \mathbb{Q}_p$ is not isomorphic to $\mathrm{M}_2(\mathbb{Q}_p)$. Fix once and for all a maximal order $\mathcal{O}_D$ of D.

Let $R = \varprojlim (\mathbb{Z}/M\mathbb{Z})$, where the limit is over all positive integers M prime to d. Fix once and for all an isomorphism $\kappa\colon \mathcal{O}_D \otimes_{\mathbb{Z}} R \to \mathrm{M}_2(R)$, and note that κ induces an isomorphism $\mathcal{O}_D \otimes_{\mathbb{Z}} T \cong \mathrm{M}_2(T)$ for all quotients T of R. Write $\mathcal{O}_{D,f}$ for $\mathcal{O}_D \otimes_{\mathbb{Z}} \hat{\mathbb{Z}}$. Then κ induces a natural map $\mathcal{O}_{D,f} \to \mathrm{M}_2(\mathbb{Z}/N\mathbb{Z})$ for any positive integer N prime to d, by the composite of the maps $\mathcal{O}_{D,f} \to \mathcal{O}_D \otimes_{\mathbb{Z}} R \xrightarrow{\kappa} \mathrm{M}_2(R) \twoheadrightarrow \mathrm{M}_2(\mathbb{Z}/N\mathbb{Z})$. Finally, fix once and for all an isomorphism $\kappa_\infty\colon D \otimes \mathbb{R} \to \mathrm{M}_2(\mathbb{R})$.

Let S be a scheme on which d is invertible. A *false elliptic curve* over S is a pair $(A/S, i)$ where A/S is an abelian surface (that is, A is an abelian scheme over S of relative dimension two), and $i\colon \mathcal{O}_D \hookrightarrow \operatorname{End}_S(A)$ is an injective ring homomorphism (where throughout this paper all ring homomorphisms are assumed to

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