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ON THE REGULARITY PROPERTIES OF A MODEL PROBLEM RELATED TO WAVE MAPS

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1. Introduction. In this paper we shall consider the nonlinear wave equation

$$\square\,\phi = \partial_i[\phi, R_0 R_i \phi] \tag{1.1}$$

subject to the initial value problem,

$$\phi(0,x) = f_0(x) \in H^s(\mathbb{R}^n) \qquad \partial_t\phi(0,x) = g_0(x) \in H^{s-1}(\mathbb{R}^n). \tag{1.2}$$

Here ϕ is a scalar function defined on Minkowski space-time $\mathbb{R}^{n+1}$ with values in a fixed Lie algebra of matrices endowed with the standard Lie bracket $[\phi, \psi] = \phi \cdot \psi - \psi \cdot \phi$. The operators R_0, R_i are the nonlocal operators $R_0 = (-\Delta)^{-1/2}\partial_t$, $R_i = (-\Delta)^{-1/2}\partial_i$. The equation (1.1) arises in connection to the equation

$$\partial_\alpha A_\beta - \partial_\beta A_\alpha = [A_\alpha, A_\beta], \tag{1.3a}$$

$$\partial^\alpha A_\alpha = 0, \tag{1.3b}$$

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where A_α is a connection one-form defined on $\mathbb{R}^{n+1}$. Indeed, observe that (1.3a), (1.3b) imply

$$\square A_0 = \partial^i[A_i, A_0].$$

On the other hand, if we set

$$\bar{A}_i = A_i + (-\Delta)^{-1}\partial_0\partial_i A_0 = A_i + R_0 R_i A_0, \tag{1.3c}$$

then, in view of (1.3b),

$$\partial^i \bar{A}_i = \partial^i A_i - \partial_0 A_0 = \partial^\alpha A_\alpha = 0,$$

and, according to (1.3a),

$$\partial_i \bar{A}_j - \partial_j \bar{A}_i = [\bar{A}_i - R_0 R_i A_0, \bar{A}_j - R_0 R_j A_0].$$

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