Vol. 87, No. 3 DUKE MATHEMATICAL JOURNAL © 1997

HARMONIC MEASURE ON LOCALLY FLAT DOMAINS

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1. Introduction. The basic aim of this paper is to study the relationship between the harmonic measure of a domain and the geometry of its boundary. We concentrate on domains whose boundary is *locally flat*, where this notion will be understood in a weak sense. Let $\Omega \subset \mathbf{R}^{n+1}$ be an open set. Loosely speaking, we say that $\partial\Omega$ is locally flat if locally it can be well approximated by affine spaces. In particular, we will see that in this case, Ω is a non-tangentially accessible domain, and therefore its harmonic measure ω is doubling (see [JK1]). We prove that if $\partial\Omega$ is well approximated by n-planes in the Hausdorff-distance sense, then the doubling constant of ω asymptotically approaches the doubling constant of the n-dimensional Lebesgue measure. If, moreover, the unit normal vector to $\partial\Omega$ has small mean oscillation (i.e., small BMO norm), then the Poisson kernel behaves like the Poisson kernel on Lipschitz domains with small constant. From this we conclude that if the unit normal vector to $\partial\Omega$ has vanishing mean oscillation (i.e., it is in VMO), then the Poisson kernel behaves like the Poisson kernel on C' domains. In order to be more specific, we need to introduce some definitions.

Definition 1.1. Let $\Omega \subset \mathbf{R}^{n+1}$; we say that $\partial\Omega$ separates $\mathbf{R}^{n+1}$ if there exist $\delta > 0$ and $R > 0$ such that for each $Q \in \partial\Omega$, there exist an n-dimensional plane $\mathscr{L}(R,Q)$ containing Q and a choice of unit normal vector to $\mathscr{L}(R,Q)$, $n_{R,Q}$ satisfying

$$\mathscr{T}^+(R,Q) = \{X = (x,t) = x + tn_{R,Q} \in B^{n+1}(R,Q) \colon x \in \mathscr{L}(R,Q), t > 2\delta R\} \subset \Omega, \tag{1.1}$$

and

$$\mathscr{T}^-(R,Q) = \{X = (x,t) = x + tn_{R,Q} \in B^{n+1}(R,Q) \colon x \in \mathscr{L}(R,Q), t < -2\delta R\} \subset \Omega^c. \tag{1.2}$$

Here $B^{n+1}(R,Q)$ denotes the $(n+1)$-dimensional ball of radius R and center Q.

Definition 1.2. Let $\Omega \subset \mathbf{R}^{n+1}, \delta > 0, R > 0$. We say that Ω is a (δ, R)-*Reifenberg-flat domain* if $\partial\Omega$ separates $\mathbf{R}^{n+1}$, and for each $Q \in \partial\Omega$, and for every $r \in (0,R]$,

Received 1 November 1995. Revision received 23 May 1996.
Kenig's work partially supported by the National Science Foundation.
Toro's work partially supported by a National Science Foundation postdoctoral fellowship.