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# ON AN ELLIPTIC ANALOGUE OF ZAGIER'S CONJECTURE

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**Introduction.** This article is the elliptic version of "Interprétation motivique de la conjecture de Zagier reliant polylogarithmes et régulateurs" [BD]. The object of this work is a conjecture which predicts that certain formal linear combinations of elements of the Mordell-Weil group are homologically meaningful, i.e., yield elements in certain $K$-groups of symmetric powers of elliptic curves. It also predicts their images under the Beilinson regulator. We refer to this as the *weak version of Zagier's conjecture for elliptic curves.*

The formulation of the conjecture is contained in §1. The most general context in which it can be stated is that of *families* of elliptic curves over a base $B$. However, we give the statement for the image under the regulator only when $B$ is the spectrum of a number field, where Deligne cohomology admits an easy description. Since we require functoriality with respect to pullbacks, this does still impose nontrivial restrictions on the regulator for arbitrary bases.

We remark that if the formalism predicted by the conjecture satisfied a certain surjectivity requirement, then Beilinson's conjecture would give a description of values of $L$-functions of symmetric powers of elliptic curves in terms of determinants of values of Eisenstein-Kronecker series.

For $CM$-elliptic curves satisfying Shimura's condition $(S)$, this relation was shown in [De1] and [De2]. For $L(E, 2)$, which in our notation corresponds to the case $k = 3$, it has recently been proven for modular elliptic curves [GL].

In § 2 and § 3, we develop a machinery that will allow us to construct extensions from certain formal linear combinations of elements in the Mordell-Weil group, as soon as we work in a category of smooth sheaves satisfying certain axioms, among which is the existence of elliptic polylogarithmic extensions.

In § 4, we show that if a category of mixed motivic sheaves with these axioms exists, and if it has the right Ext-groups, then the weak version of Zagier's conjecture follows from the conjunction of the results of § 3, and of the description of the Hodge version of the elliptic polylogarithm given elsewhere.

This work would not exist were it not for N. Schappacher, who on more than one occasion insisted with vigour that the author concern himself with the elliptic Zagier conjecture. I thank him warmly.

Conversations with him and with S. Bloch and A. J. Scholl helped me realize that an integrality criterion should be included in the conjecture. However, its sheaf-theoretic interpretation remains a desideratum. The special shape of the

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