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A TRACE FORMULA FOR DUAL PAIRS

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1. Introduction. Let F be a number field and $\mathbf{A}$ its adele ring. Let G' be a simple, split, simply laced group defined over F. Assume that G' is not of type A_n. There is a dual pair of reductive groups G and SL_2 in G'; i.e., $G \times SL_2$ embeds into G', with G and SL_2 satisfying the double centralizer property in G'. In this paper, we propose a study on the correspondence between the automorphic forms on $G(\mathbf{A})$ and the automorphic forms on $SL_2(\mathbf{A})$ using the trace formula.

Let Δ be the set of simple roots of G'. Let $\tilde{\alpha}$ be a highest-weight root of G'. Let α be the unique simple root that is not perpendicular to $\tilde{\alpha}$. Let P' be the maximal parabolic subgroup of G' corresponding to $\Delta\backslash\{\alpha\}$, with $P' = M'U'$ being its Levi decomposition. Then U' is a Heisenberg group with a one-dimensional center Z. The quotient space U'/Z has the structure of a symplectic vector space. The adjoint action of the group M' on U' factors to a map $j\colon M' \to GSp(U'/Z)$. Let G be the derived group of M'. Then $j(G)$ lies in $Sp(U'/Z)$, and (G, SL_2) is a dual pair in G'. If G' is of type E_i, $i = 6, 7, 8$, or D_m, then G is semisimple split of the type A_5, D_6, E_7, or $A_1 \times D_{m-2}$.

The correspondence between the automorphic forms on $G(\mathbf{A})$ and $SL_2(\mathbf{A})$ can be studied using the *theta kernel*. For the group G', one can construct an automorphic θ-module π (see [GRS2]). Here $\pi = \bigotimes \pi_v$, where each local component π_v is of class-one and of the smallest Gelfand-Kirillov dimension. At a finite place v, the representation π_v is the unique minimal representation constructed in [K], [KS], and [S]. If v is Archimedean, such a representation π_v is considered in [V]. The θ-module is an analogue of the θ-representation for the twofold cover of Sp_n constructed by A. Weil [W]. As is the case for Weil's construction, the θ-module can be realized in the residue spectrum of $L^2(G'(F)\backslash G'(\mathbf{A}))$. For any dual pair (H_1, H_2) in G', restricting the θ-module to $H_1 \times H_2$ gives a θ-correspondence between the automorphic forms on H_1 and these on H_2.

Our trace formula approach will not make use of the θ-module. However, the trace formula is directly inspired by the results on θ-correspondence. We summarize some of the works done on the θ-correspondence in the case G' is of type G_2 (not simply laced); see [GRS1]. Here the θ-module is an automorphic representation of $\tilde{G}'$, the threefold cover of G', constructed by Savin in [S]. The restriction of the θ-module to the dual pair $SL_2 \times \widetilde{SL}_2^3$ (threefold cover for the second component) produces a decomposition of the form $\bigoplus \pi \otimes \theta(\pi)$. Here π and $\theta(\pi)$ are cuspidal representations of $SL_2(\mathbf{A})$ and $\widetilde{SL}_2^3(\mathbf{A})$, respectively, with

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