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BOUNDED ORBITS OF ANOSOV FLOWS

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1. Introduction. In this paper we develop a symbolic dynamics approach to a problem of studying dimensional characteristics of the set of bounded geodesics on manifolds of negative curvature.

Recall that an orbit of a flow on a noncompact manifold is called *bounded* if it is confined to a compact set. A *bounded geodesic* is a bounded orbit of the geodesic flow.

The problem under consideration goes back to the classical theorem of Jarnik and Besicovitch, which states that the set of badly approximable numbers on the segment $[0, 1]$ has Hausdorff dimension equal to 1. Recall that a real number x is called *badly approximable* if for any rational p/q one has $|x - p/q| > C(x)/q^2$. The equivalent definitions are the following: x is badly approximable if the partial convergents of its continued fraction $k_j(x)$ are bounded or if the closure of the x-orbit by the Kuzmin-Gauss map $x \to \{1/x\}$ does not contain 0.

The above-mentioned result can be reformulated in another way. Consider the modular surface M on the Poincaré model of the Lobachevsky plane. A geodesic on M is bounded if and only if both its endpoints are badly approximable. Thus the Jarnik-Besicovitch theorem is equivalent to the fact that the Hausdorff dimension of the set of bounded geodesics on the unit tangent bundle to M equals 3.

This result was generalized to the manifolds of the constant negative curvature having finitely generated fundamental groups by Patterson [Pt], Stramann [St], Fernandez and Melian [FM], and Bishop and Jones [BJ], and to cofinite manifolds of (variable) nonpositive curvature by Dani [D1], [D2]. In all cases considered, the Hausdorff dimension of the set of bounded geodesics is equal to the Hausdorff dimension of the set of recurrent geodesics, i.e., those whose forward and backward rays each spend an infinite time in some compact region.

Note that the set of bounded geodesics has zero Liouville-Patterson measure by the ergodicity of the geodesic flow. However, the above-mentioned statement is not so surprising because geodesic flows on manifolds of negative curvature have an abundance of invariant measures so that the set of nontypical points for any of them is quite large.

Another generalization of the Jarnik-Besicovitch theorem comes from the third definition of badly approximable number. Namely, one may ask whether, for an arbitrary Anosov system on a compact manifold, it is true that the dimension of the set of orbits whose closure does not contain some point (or a

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