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HÖLDER FOLIATIONS

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1. Introduction. It is the goal of this paper to estimate the regularity of the holonomy maps of certain dynamically invariant foliations. They are θ-Hölder. This Hölder regularity is a crucial component of the analysis appearing in our papers, Pugh and Shub [12] and Wilkinson [16], where we establish stable ergodicity for a wide class of dynamical systems, a class which includes Anosov diffeomorphisms, the time t-maps of many Anosov flows, and many examples defined on homogeneous spaces of Lie groups. We state here our main results, place them in context, and then go on to explain them more fully in §2. The notation $m(T)$ stands for the *conorm* (or *minimum norm*) of a linear transformation, $m(T) = \|T^{-1}\|^{-1}$.

THEOREM A. *Suppose that $f: M \to M$ is a C^2 diffeomorphism, partially hyperbolic with respect to the splitting $TM = E^u \oplus E^c \oplus E^s$. Then, for some $\theta \in (0, 1)$ and all $p \in M$, its expansion and contraction rates satisfy a θ-*pinching condition

$$\|T_p^s f\| \, \|T_p^u f\|^\theta < m(T_p^c f) \qquad \text{and} \qquad \|T_p^c f\| < m(T_p^u f) m(T_p^s f)^\theta.$$

For any such θ, the local unstable and stable holonomy maps are uniformly θ-Hölder.

THEOREM B. *Suppose that $f: M \to M$ is a partially hyperbolic C^2 diffeomorphism and f leaves invariant a foliation $\mathscr{W}^c$ tangent to the center direction E^c. (The tangent plane to the $\mathscr{W}^c$-leaf at p is E_p^c.) If the expansion and contraction rates satisfy the* center bunching conditions

$$\|T_p^s f\| \, \|T_p^c f\| < m(T_p^c f) \qquad \text{and} \qquad \|T_p^c f\| < m(T_p^u f) m(T_p^c f),$$

then the local unstable and local stable holonomy maps are uniformly C^1 when restricted to each center unstable and each center stable leaf, respectively.

A C^2 volume-preserving diffeomorphism of a compact, connected manifold $M \to M$ is *stably ergodic* if it and all its C^2 small volume-preserving perturbations are ergodic. In 1962 Anosov [2] proved that totally hyperbolic diffeomorphisms are stably ergodic. By contrast, the theory of Kolmogorov, Arnold, and Moser produces open sets of nonergodic diffeomorphisms that have no hyperbolicity at all. In a series of recent papers, we have been studying the mixed situation, in which the dynamical system is partially, but not totally, hyperbolic. Our main

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