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HALF-DENSITY VOLUMES OF REPRESENTATION SPACES OF SOME 3-MANIFOLDS AND THEIR APPLICATION

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§1. Introduction. In this paper we compute half-density volumes of the irreducible $SU(2)$-representation spaces of Seifert fibred manifolds and graph manifolds. The half-density over the irreducible $SU(2)$-representation space of a 3-manifold comes from the Reidemeister torsion for Ad-$SU(2)$-representation. More precisely, the determinant term of the first homology of the Reidemeister torsion gives the half-density of the irreducible representation space. This is because the tangent space of the irreducible representation space can be identified with the first cohomology of the twisted cochain complex.

The motivation of this paper is given by two sources. The first source is E. Witten's method to compute the symplectic volume of the irreducible $SU(2)$-representation space of a Riemann surface. In [W2] Witten suggested a useful method to compute the symplectic volume of this space using the Reidemeister torsion and the character theory of the Lie group $SU(2)$.

The second source is the invariant defined by L. C. Jeffrey and J. Weitsman in [JW1]. This invariant is motivated by the asymptotic expansion of the Witten invariant of a 3-manifold. Jeffrey and Weitsman define this invariant using the Reidemeister torsion as a half-density measure of the irreducible $SU(2)$-representation space. Hence, to compute this invariant we must compute the Reidemeister torsion completely, including the determinant term of the homology, which is the half-density measure of the $SU(2)$-representation space. In this paper we call this invariant the Jeffrey-Weitsman-Witten invariant.

From the above motivations, we could consider naturally that this invariant might be computed by the method of [W2]. The examples to which we apply the method of Witten are Seifert fibred manifolds and graph manifolds. This is because these manifolds are made from the trivial circle bundle over the Riemann surfaces by twisting finite fibres. So the method of Witten is applicable with some modification.

To compute the half-density derived from the Reidemeister torsion, we must compute both the scalar part and the determinant part of the Reidemeister torsion. The method to compute the scalar part comes from [F]. For Seifert fibred manifolds and graph manifolds, this value gives the weight of the half-density to each connected component of the irreducible $SU(2)$-representation space. So we combine two methods of [F] and [W2] to compute the half-density volumes of

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