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PERVERSE SHEAVES AND QUIVERS

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Many mathematical questions can be reduced to questions about the category of perverse sheaves on a complex algebraic variety X (see [Lu2] for a survey). One is usually interested in the subcategory $P_\Lambda(X)$, consisting of perverse sheaves whose characteristic variety is contained within a fixed conical Lagrangian subvariety $\Lambda \subset T^*X$ of the cotangent bundle of X.

In this paper, we show that $P_\Lambda(X)$ is equivalent to a category of finite quivers. This is a strong finiteness condition on $P_\Lambda(X)$ (see Section 0 for an explanation). In particular, it implies that this category is equivalent to the category of finite-dimensional modules over a finitely presented algebra, a finiteness property of categories of perverse sheaves that does not appear to be evident from other approaches. For some particular pairs (X, Λ), the quiver description of the category $P_\Lambda(X)$ was already known (see [GKh], [GGM], [Kh], [MV2], [S] and the references therein).

Given X and Λ, we explicitly construct (in Definition 6.3) a quiver category which is equivalent to $P_\Lambda(X)$. The construction is geometric. By an appropriate embedding and a contact transformation, we are reduced to the case where X is projective N space, and Λ is the conormal variety to a hypersurface $Y \subset X$. Picking a point $p \in X$ not on Y and a line $L \subset X$ through p, we are led to classifying the category $P_{\Lambda'}(L - p)$ where Λ' is the conormal variety to the finite collection of points $L \cap Y$. Most of the paper concerns $P_\Lambda(L - p)$, its localization at the constant sheaves, and its variation in families. Applying this work to the family of lines L through p gives the result.

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0. Finite quiver categories. A *quiver type* is a finite directed graph, with vertices $v_1, \ldots, v_n$ and edges $e_0, \ldots, e_m$ such as

$$v_1 \underset{e_2}{\overset{e_1}{\leftrightarrows}} v_2 \underset{e_4}{\overset{e_3}{\rightleftarrows}} v_3.$$

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