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THE GROUP ACTION ON THE CLOSED FIBER OF THE LUBIN-TATE MODULI SPACE

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0. Introduction. Let G_0 be a one-dimensional formal group of height $h < \infty$ over an algebraically closed field k of characteristic $p > 0$. It is known that the deformations of G_0 are unobstructed. The universal deformation space of G_0 is isomorphic to $\mathrm{Spf}\, W(k)[[u_1, \ldots, u_{h-1}]]$, and the equicharacteristic deformation space of G_0 is isomorphic to $\mathrm{Spf}\, k[[u_1, \ldots, u_{h-1}]]$. The group $\mathrm{Aut}(G_0)$ of automorphisms of G_0 is isomorphic to the group of units $\mathcal{O}_B^\times$ of the maximal order $\mathcal{O}_B$ in the central division algebra $B = B_{1/h}$ over $\mathbb{Q}_p$ with invariant $1/h$. There is a natural action of $\mathrm{Aut}(G_0)$ on $\mathrm{Spf}\, W(k)[[u_1, \ldots, u_{h-1}]]$ and $\mathrm{Spf}\, k[[u_1, \ldots, u_{h-1}]]$ "by changing the marking of the closed fiber." Even though the deformation space looks simple, the action of the compact p-adic group $\mathrm{Aut}(G_0)$ on it is extremely complicated. For instance, the equivariant cohomologies of some naturally defined vector bundles contain deep information about the stable homotopy theory. Very little computation of these equivariant cohomology groups has been done. This action itself remains a mystery in most aspects.

Much insight about this action comes from the p-adic period map

$$\rho\colon \mathrm{Spf}\, W(k)[[u_1, \ldots, u_{h-1}]] \otimes \mathrm{frac}(W(k)) \to \mathbb{P}^{h-1}_{\mathrm{frac}(W(k))},$$

which was described explicitly in [GH]. Here $\mathrm{Spf}\, W(k)[[u_1, \ldots, u_{h-1}]] \otimes \mathrm{frac}(W(k))$ denotes the rigid open unit ball, thought of as the generic fiber of $\mathrm{Spf}\, W(k)[[u_1, \ldots, u_{h-1}]]$. The p-adic period map comes from comparing the position of the Hodge filtration relative to the space of horizontal sections of the covariant isocrystal attached to G_0. The target space is the projective space of one-dimensional quotients of the covariant isocrystal attached to G_0; therefore, $(\mathrm{End}(G_0) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p)^\times$ operates on it via a natural linear representation on the isocrystal. It is a pleasant fact that the period map ρ is étale and surjective [GH], [Laf]. The period map ρ is equivariant with respect to the inclusion $\mathrm{Aut}(G_0) \subset (\mathrm{End}(G_0) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p)^\times$, so the complicated action of $\mathrm{Aut}(G_0)$ is "linearized" by the period map ρ. However, the period map is far from being defined over $W(k)$. This makes it extremely difficult to extract information about the action of $\mathrm{Aut}(G_0)$ on the closed fiber $\mathrm{Spf}\, k[[u_1, \ldots, u_{h-1}]]$.

This article is a report on my attempt to study the orbit structure of this action, after simplifying the problem by taking the Zariski closure of the orbits. On the

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