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NEW CHANNELS OF SCATTERING FOR THREE-BODY QUANTUM SYSTEMS WITH LONG-RANGE POTENTIALS

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1. Introduction. An aim of the scattering theory is to find the asymptotics as $t \to \infty$ of the solution $u(t) = \exp(-iHt)f$ of the time-dependent Schrödinger equation with a Hamiltonian $H = -2^{-1}\Delta + V(x)$, $V(x) = \overline{V(x)}$, in the space $\mathscr{H} = L_2(\mathbb{R}^d)$. If f is an eigenvector of H, i.e., $Hf = \lambda f$, then obviously $u(t) = \exp(-i\lambda t)f$. Suppose now that f is orthogonal to the subspace $\mathscr{H}^{(b)}$ spanned by all eigenvectors. In the two-body short-range case when $V(x) = O(|x|^{-\rho})$, $\rho > 1$, the asumptotics of $u(t)$ is the same as that for the free system, that is,

$$\exp(-iHt)f = \exp(-iH_0t)f_0 + o(1) \tag{1.1}$$

for some $f_0 \in \mathscr{H}$ and $H_0 = -2^{-1}\Delta$. The symbol $o(1)$ means a function such that its norm in the space $\mathscr{H}$ tends to zero as $t \to \infty$. One can rewrite (1.1) in an equivalent way as

$$(\exp(-iHt)f)(x) = \exp(i\Phi_0(x, t))t^{-d/2}g(x/t) + o(1), \tag{1.2}$$

where $\Phi_0(x, t) = x^2(2t)^{-1}$, $g = \exp(i\pi d/4)\hat{f}_0$ and $\hat{f}_0 = Ff_0$ is the Fourier transform of f_0.

The relation (1.2) also holds true (see, e.g., [1]) for long-range potentials satisfying the condition

$$|D^\kappa V(x)| \leqslant C(1 + |x|)^{-\rho-|\kappa|}, \qquad \rho > 0, \tag{1.3}$$

for $|\kappa| = 0, 1, 2$. In this case, the phase function $\Phi_0(x, t)$ is a (perhaps, approximate) solution of the eikonal equation

$$\partial\Phi_0/\partial t + 2^{-1}|\nabla\Phi_0|^2 + V = 0. \tag{1.4}$$

The asymptotics (1.2) show that, if f belongs to the absolutely continuous subspace $\mathscr{H}^{(ac)} = \mathscr{H} \ominus \mathscr{H}^{(p)}$ of the operator H, then the solution $(\exp(-iHt)f)(x)$ "lives" in the region where $|x| \sim t$. The mapping W_0: $g \mapsto f$ determined by (1.2) is isometric and its range coincides with the subspace $\mathscr{H}^{(ac)}$. Clearly, W_0F is the usual (modified) wave operator relating H_0 and H.

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