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MULTIPLICITY g POINTS ON THETA DIVISORS

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Let Θ denote the theta divisor of a principally polarized abelian variety (ppav) A over the complex numbers. Let $g = \dim(A)$ and $\mathrm{Sing}_m(\Theta) = \{p \in \Theta \mid \mathrm{mult}_p(\Theta) \geqslant m\}$. J. Kollár [K, Theorem 17.13] proved that the pair (A, Θ) is log canonical, hence $\dim(\mathrm{Sing}_m(\Theta)) \leqslant g - m$ for each $m \geqslant 2$. In particular, for $m = g + 1$, $\mathrm{Sing}_{g+1}(\Theta) = \varnothing$; that is, the theta divisor cannot have a point of multiplicity larger than g. Kollár also remarked [K, Remark 17.13.1] that it may be possible that if $\dim(\mathrm{Sing}_m(\Theta)) = g - m$ for some $2 \leqslant m \leqslant g$, then (A, Θ) is a product of lower-dimensional ppav's. (The $m = 2$ case of the question was already well known as the "N_{g-2} conjecture.") In this paper, we settle affirmatively the case of multiplicity $m = g$.

THEOREM. *Let (A, Θ) be a g-dimensional ppav over the complex numbers. Suppose that there exists a point of multiplicity g on the theta divisor Θ. Then (A, Θ) is isomorphic (as ppav) to the product of g elliptic curves. In particular, if (A, Θ) is indecomposable (equivalently, if Θ is irreducible), then* $\mathrm{Sing}_g(\Theta) = \varnothing$.

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The idea of the proof is to exploit the relation between the self-intersection number of the cohomology class of Θ and the multiplicities of certain geometric self-intersections of Θ. Briefly, under the assumption that Θ has a point of multiplicity g, a 1-cycle C is constructed in the self-intersection class $[\Theta]^{(g-1)}$ (using Kollár's theorem), such that $C = (g-1)!\Gamma$, where Γ is an effective 1-cycle representing the cohomology class $\theta^{g-1}/(g-1)!$ By the Matsusaka-Hoyt criterion,

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