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SINGULAR SCHEMES OF HYPERSURFACES

PAOLO ALUFFI

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0. Introduction. Let Y be the singular locus of a hypersurface X in a smooth variety M, with the scheme structure defined by the Jacobian ideal of X. (We will say then that Y is the singular *scheme* of X, to emphasize that the scheme structure of Y is important for our considerations.) In this note, we consider a class in the Chow group of Y which arises naturally in this setup and which captures much intersection-theoretic information about the situation. The guiding question we have in mind is: which schemes Y can arise as singular schemes of hypersurfaces? We will obtain strong constraints showing, for instance, that the only hypersurfaces in $\mathbb{P}^N$ whose singular *schemes* are positive-dimensional linear subspaces of $\mathbb{P}^N$ are quadrics, and that no (reduced) nodal curve can be the singular scheme of a hypersurface in a nonsingular variety. Many more statements of this sort can be found in §3.

A different type of application is in §2: we show how our class relates to other invariants of the singularity of a hypersurface; the class can be used to recover results of Holme and Parusiński on degree and multiplicity of dual varieties, and leads naturally to a generalization of the notion of "ranks" of a (smooth) projective variety. Also, we obtain a strengthening of Landman's parity result, and a new proof of a result of Zak on the dimension of the dual of a smooth variety. The duality results follow by applying the framework to hyperplane sections of M: the singular scheme of a section is supported on the locus of contact of the hyperplane with M, and the class can be used to measure this contact. For example, the class measures how "general" a given section is: we show (Corollary 2.6) that if the contact scheme is a linear subspace $\mathbb{P}^{r-1}$, then the corresponding hyperplane is a smooth point of the dual variety of M, and the dual variety has codimension r.

The main general results are in §1, where we prove (Corollary 1.7) that the class we introduce depends in fact only on Y and on the line bundle $\mathscr{L} = \mathscr{O}(X)|_Y$, and not on the ambient variety M (provided that Y is the singular scheme of a section

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