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# THEORY OF TUBULAR NEIGHBORHOOD IN ETALE TOPOLOGY

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**0. Introduction.** In his celebrated paper [D], Drinfel'd established a nonabelian reciprocity law for $GL(2)$ over a function field. In the proof, etale cohomology groups for rigid analytic spaces were defined and calculated for the $p$-adic symmetric upper half plane. In fact, he defined it in an ad hoc manner, though the general definition of rigid etale cohomology theory is not so difficult [FV]. But one needs GAGA-type comparison theorems in various stages, for example Schneider and Stuhler [SS] calculated the cohomology of $p$-adic symmetric spaces assuming the comparison statement which will be made explicit in Section 6. Moreover, such theorems are necessary even in the study of Shimura varieties [Car].

Our aim in this paper is to establish basic comparison theorems for general rigid analytic spaces in Raynaud's sense [Ray2], especially for torsion etale sheaves on proper schemes over affinoid spaces, defined by noetherian rings or topologically finitely generated rings over valuation rings.

Because of the lack of a foundation of general rigid geometry, basic properties are included in §4. Especially, we introduce a topological space associated to a rigid analytic space, which enables us to avoid the use of the Grothendieck topology using admissible coverings.

To assure a continuity for filtered projective limit (5.2.1), we need to use henselian schemes ([Gre1], [Gre2], [Cox]) instead of formal schemes and "rigid geometry" for henselian schemes. Basic properties for it are proved in the same way as rigid analytic geometry in §4, and rigid-etale topology is defined for henselian schemes too. Using Elkik's approximation theorems, we know that the rigid-etale toposes are the same for henselian schemes and their completions (5.1.3), and the comparison statement is reduced to the proper base change theorem for schemes. By this method the claim follows for henselian local rings. For the general case, Gabber's base change theorem [Ga] is used.

Our proof leads to the comparison theorem for nonabelian coefficients too, which is briefly mentioned in this paper.

As an application to etale cohomology of schemes, the regular base change theorem, conjectured in [AGV3, XIX], is proved in §7.

Concerning etale topology for analytic spaces, Berkovich [Ber] established basic properties of etale cohomology for his analytic spaces (not rigid analytic ones, and the topology is different from ours), and the comparison statement is

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