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COHOMOLOGY OF DYNAMICAL SYSTEMS AND RIGIDITY OF PARTIALLY HYPERBOLIC ACTIONS OF HIGHER-RANK LATTICES

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1. Introduction. Let $SL(n, \mathbb{Z})$ be the group of $n \times n$ integer matrices of determinant one, and let π be the standard action by linear automorphisms of $SL(n, \mathbb{Z})$ on the torus $\mathbb{T}^n$. Let d be a positive integer, and consider the action ρ_0 of $SL(n, \mathbb{Z})$ on $\mathbb{T}^{n+d} = \mathbb{T}^n \times \mathbb{T}^d$ given by $\rho_0(A)(x, y) = (\pi(A)x, y)$ (where $x \in \mathbb{T}^n$, $y \in \mathbb{T}^d$, and $A \in SL(n, \mathbb{Z})$). Then ρ_0 is a (nonirreducible and nontransitive) partially hyperbolic action.

This paper emerges from an attempt to understand the differentiable actions close to ρ_0. After Zimmer initiated the program to classify volume-preserving actions of higher-rank lattices on compact manifolds (see [Z1], [Z2]); Hurder, Katok, Lewis, Qian, Spatzier, Zimmer, and others tried to understand local, global, and infinitesimal rigidity of these actions (see [H1], [H2], [KL1], [KL2], [KLZ], [Q1], [Q2], [Z3]). In the great majority of these papers, an essential use was made of the presence of an Anosov element. Here we show for the first time the deformation rigidity of nonirreducible and nontransitive partially hyperbolic actions (for terminology, see §7). One example is ρ_0, for $n \geqslant 3$ and any integer d; others are given in §7. Precise statements are Theorems 7.1 and 7.8.

As turns out from this study, there is a close connection between the rigidity of the natural actions of lattices on tori and the cohomology of hyperbolic dynamical systems. The first nontrivial results about this cohomology were obtained by Livsic and Sinai in [L1], [L2], and [LS]. We recall some definitions and Livsic's main results.

Let G be a discrete group acting on a closed Riemannian manifold M, and let Γ be some topological group with unit 1_Γ. A cocycle β is a continuous function $\beta: G \times M \to \Gamma$ such that

$$\beta(g_1 g_2, x) = \beta(g_1, g_2 x)\beta(g_2, x),$$

for all $g_1, g_2 \in G$, $x \in M$. Two cocycles β_1 and β_2 are called cohomologous if there exists a continuous map $P: M \to \Gamma$ such that

$$\beta_1(g, x) = P(gx)\beta_2(g, x)P(x)^{-1},$$

for all $g \in G$, $x \in M$.

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