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ON THE SYMPLECTIC STRUCTURE OF THE PHASE SPACE FOR PERIODIC *KdV*, TODA, AND DEFOCUSING NLS

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1. Introduction and summary of results. It is well known that the Toda lattice $(1 \leqslant n \leqslant N)$, $N \geqslant 3$,

$$\dot{Q}_n = P_n, \qquad \dot{P}_n = e^{Q_{n-1}-Q_n} - e^{Q_n-Q_{n+1}}$$

with periodic boundary conditions, is a completely integrable Hamiltonian system. The eigenvalues of the associated Jacobi matrices

$$L^{\pm} = L^{\pm}(b, a) = \begin{bmatrix} b_1 & a_1 & 0 & \cdots & \pm a_N \\ a_1 & & & & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & & & & a_{N-1} \\ \pm a_N & \cdots & 0 & a_{N-1} & b_N \end{bmatrix}$$

are a set of conserved quantities. Here b_n and a_n are the Flaschka coordinates given by (cf. [Moe, Chapter 7])

$$b_n = -P_n, \qquad a_n = e^{(Q_n-Q_{n+1})/2} \tag{1.1}$$

with the convention that $b_n = b_{n+N}$, $a_n = a_{n+N}$. In these variables the Toda equations take the form $(\dot{b}, \dot{a}) = J\nabla_{(b,a)}H$, where H is the Hamiltonian $H = (1/8)\sum_{n=1}^N b_n^2 + (1/4)\sum_{n=1}^N a_n^2$ and J is the (degenerate) symplectic matrix

$$\begin{pmatrix} 0 & A \\ -{}^tA & 0 \end{pmatrix}$$

with A given by

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