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GENERALIZED PRINCIPAL SERIES REPRESENTATIONS AND TUBE DOMAINS

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0. Introduction. Let $D = G/K$ be an irreducible symmetric tube domain of rank r. Its Shilov boundary S is of the form $S = G/P$, where $P = MAN$ is a maximal parabolic subgroup of G, A is a Lie subgroup with one-dimensional Lie algebra $\mathfrak{a}$ and N an abelian subgroup with Lie algebra $\mathfrak{n}$. Let 2δ on $\mathfrak{a}$ be as usual the trace of the adjoint action of $\mathfrak{a}$ on $\mathfrak{n}$. The purpose of this paper is to study the composition series of the parabolically induced representations $\pi(\nu) = \mathrm{Ind}_{MAN}^{G}(1 \otimes e^{\nu} \otimes 1)$ of G on the Shilov boundary S and $\pi(\alpha, \nu) = \mathrm{Ind}_{\tilde{P}}^{\tilde{G}}(\alpha \otimes e^{\nu} \otimes 1)$ of the universal covering group $\tilde{G}$ of G for α a character. These are sometimes referred to as generalized principal series representations.

Let $H(\nu)$ be the representation space for $\pi(\nu)$. There is an operator $A(\nu)$ from $H(\nu)$ to $H(2\delta - \nu)$ intertwining the infinitesimal actions. Let $\langle\ ,\ \rangle$ be the invariant pairing between $H(\nu)$ and $H(2\delta - \nu)$. Then the invariant Hermitian form on H(ν) is given by $\langle A(\nu)f_1, f_2\rangle$.

The representation can be realized on the space $L^2(S)$ of L^2-functions on S. Under the K-action, the space $L^2(S)$ is decomposed into irreducibles with highest weights $-\mathbf{m} = -(m_1\gamma_1 + \cdots + m_r\gamma_r)$, where $\gamma_1 < \cdots < \gamma_r$ are the Harish-Chandra strongly orthogonal roots and $m_1 \geqslant m_2 \geqslant \cdots \geqslant m_r$ are integers. Our main result is an explicit formula of the eigenvalue of $A(\nu)$ on each K-irreducible and a calculation of the composition series for $H(\nu)$.

Our approach to the problem is somewhat elementary; in particular, we do not use Vogan's deep results on composition factors. Let $L = K \cap P$; then $S = K/L$ is a symmetric space of K. Each K-irreducible representation with highest weight $-(m_1\gamma_1 + \cdots + m_r\gamma_r)$ has a unique L-fixed vector, say $\phi_{-\mathbf{m}}$, also called the spherical function. Let $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$ be the Cartan decomposition of $\mathfrak{g}$ and let $\mathfrak{a} = \mathbb{R}X$. Of course, by definition $X \in \mathfrak{p}$ is an L-fixed vector. Therefore $\pi(\nu)(X)\phi_{-\mathbf{m}}$, the action of $\pi(\nu)(X)$ on the L-fixed vector $\phi_{-\mathbf{m}}$ is also L-fixed, and it is thus expanded as a linear combination of $\phi_{-\mathbf{m}'}$. We find in this paper all the coefficients in the expansion (Theorems 2.2 and 6.1). Our main result then follows from this expansion. We also derive the similar results for the general line bundle case $\pi(\alpha, \nu)$.

When, roughly speaking, $\nu(X)$ is an integer, the intertwining operator is given by a K-invariant differential operator, called the Cayley operator in [KS2]. The formula for the eigenvalues of the operator is then the Capelli identity! This has been recently obtained by Kostant-Sahi [KS1]. See also [HT], [Ho] and references therein.

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