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THE GRIFFITHS INFINITESIMAL INVARIANT FOR A CURVE IN ITS JACOBIAN

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Many interesting and deep results on algebraic cycles have been obtained by M. Green [14] and C. Voisin [23] by means of a refined version of a beautiful infinitesimal invariant of normal functions introduced by Griffiths [15]. Our purpose here is to study the Griffiths invariant for the normal function given by the Abel-Jacobi image of the basic cycle $C_+ - C_-$ in the Jacobian $J(C)$ of a curve C when C varies in moduli. We show that the invariant can be computed almost explicitly and that it carries a rich amount of information. We recover Ceresa's theorem, to the effect that the basic cycle is not in general algebraically equivalent to zero in $J(C)$ [6], and we prove that in genus three the infinitesimal invariant determines the curves; indeed, when properly interpreted, the infinitesimal invariant is the equation of the canonical model of the curve in the plane. At this point we should recall a question of Griffiths [15] page 309, which has been one motivation for our work: can a general quintic threefold X be reconstructed from the infinitesimal invariants associated with the lines on X? We believe that our results give evidence that this Torelli-like problem should have a positive answer; indeed, the quintic threefold and the Jacobian of genus three behave quite similarly with respect to the Abel-Jacobi map for cycles of dimension one (cf. [4], [7], [18], and [23]).

By studying further the behaviour of the infinitesimal invariant we prove that if C varies in some convenient nonhyperelliptic subvariety X of the moduli space M_g, then for the general curve in X the cycle $C_+ - C_-$ is not algebraically equivalent to zero. For instance, this is the case if X is a subvariety of codimension less than $(g + 2)/3$, $g > 3$, or if X is the locus of plane curves. In genus three we show that the property holds for every family X of dimension at least four which is not contained in the hyperelliptic divisor; cf. Clemens [8], page 299.

Our computation of the infinitesimal invariant was applied in [21] to prove the bound $a(a - 1) < 2g$ for the genus g of curves lying on a general abelian variety of dimension $a > 3$.

The same trend of ideas which we have employed for the Ceresa cycle can be used to study a certain basic cycle for the Quillen group $H^{g-1}(J(C), K_g)$ on

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