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ON SPIN L-FUNCTIONS FOR ORTHOGONAL GROUPS

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In this paper we study the analytic properties of partial Spin L-functions for the groups GSO_{10} and GSO_{12}. More precisely, let π be a cusp form of $GSO_{2n}(\mathbb{A})$. Write $\pi = \otimes \pi_v$ where the product is over all places of a global field F and let ω_π denote the central character of π. From general theory we can associate to each unramified representation π_v a semisimple conjugacy class t_v in the L group of $GSO_{2n}(F_v)$ which is $G\ \mathrm{Spin}_{2n}(\mathbb{C})$. Let Spin denote the 2^n (even or odd)-dimensional irreducible Spin representation of $G\ \mathrm{Spin}_{2n}(\mathbb{C})$. Given a unitary Hecke character $\chi = \otimes \chi_v$ we may form the local L-function

$$L_v(\pi_v \otimes \chi_v, \mathrm{Spin}, s) = \det[I - \mathrm{Spin}(t_v)\chi_v(p)q^{-s}]^{-1},$$

where I is the 2^n identity matrix (see Sections 3.1 and 3.2 for notations).

Let S be a finite set of places in F including the archimedean ones, such that π_v and χ_v are unramified outside S. We define the partial Spin L-function

$$L_S(\pi \otimes \chi, \mathrm{Spin}, s) = \prod_{v \notin S} L_v(\pi_v \otimes \chi_v, \mathrm{Spin}, s).$$

The aim of this paper is to show that $L_S(\pi \otimes \chi, \mathrm{Spin}, s)$ is entire for all characters χ and all *generic* cusp forms π on $GSO_{10}(\mathbb{A})$ or $GSO_{12}(\mathbb{A})$, except in the $GSO_{12}(\mathbb{A})$ case when $\omega_\pi \chi^2 \neq 1$ but $\omega_\pi^2 \chi^4 = 1$. In that case the partial L-function can have at most a simple pole at $s = 0$ or $s = 1$.

To do so we use the Rankin-Selberg method. The global constructions are different in both cases. In fact, the global integral in the case of GSO_{10} is a Hecke-type integral; thus it is an entire function of s. The construction for the GSO_{12} case uses the Siegel-Eisenstein series of GSp_6. The poles and residues of this function are well known ([10] and [7]).

We also mention that these two L-functions were studied in [11] by means of the Fourier coefficient of Eisenstein series.

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1. Notations. (a) Let J_n denote the $n \times n$ matrix defined by

$$J_n = \begin{bmatrix} & & & 1 \\ & & 1 & \\ & \cdot^{\cdot^{\cdot}} & & \\ 1 & & & \end{bmatrix}.$$

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