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HYPERBOLIC OPERATORS WITH NON-LIPSCHITZ COEFFICIENTS

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1. Introduction. The purpose of this paper is to prove the well-posedness of the Cauchy problem for second-order strictly hyperbolic operators whose coefficients are not Lipschitz-continuous but only "Log-Lipschitz": for a function a to be Log-Lipschitz (LL for short) means

$$|a(x) - a(y)| \leqslant C|x - y|\,|\log|x - y||, \tag{1.1}$$

whenever $|x - y|$ is small (say for $|x - y| \leqslant 1/2$). We consider wave operators with LL coefficients, and we prove two different type of results. First, we obtain a well-posedness result when the coefficients are LL, and second, we deal with low regularity only in the time variable. We thus go beyond the classical well-posedness result for hyperbolic operators with Lipschitz-continuous coefficients. To justify the choice of this LL regularity, we show by the construction of a counterexample (modifying slightly Theorem 10 in [4]) that LL comes up as the natural threshold beyond which no well-posedness could be expected: the right-hand side of (1.1) cannot be replaced by

$$|x - y|\,|\log|x - y||\,\varphi(|x - y|) \qquad \text{with} \quad \varphi(r) \xrightarrow[r\to 0^+]{} +\infty$$

without ruining the existence of a distribution solution. Let us describe now the first kind of results. We are concerned with wave equations in divergence form,

$$P \equiv \frac{\partial^2}{\partial t^2} - \sum_{1 \leqslant i,j \leqslant n} \frac{\partial}{\partial x_i} a_{ij}(t, x) \frac{\partial}{\partial x_j}, \tag{1.2}$$

with a_{ij} real-valued satisfying

$$\begin{cases} a_{ij} = a_{ji} & \text{and} \\ \displaystyle\sum_{1 \leqslant i,j \leqslant n} a_{ij}\xi_i\xi_j \geqslant \delta|\xi|^2, \delta > 0 & \text{for any } \xi \in \mathbb{R}^n. \end{cases} \tag{1.3}$$

Moreover, $a_{ij} \in LL$ (isotropically), i.e., ($y_i \in \mathbb{R}^{1+n} = \mathbb{R}_t \times \mathbb{R}^n_x$)

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