Vol. 77, No. 2 DUKE MATHEMATICAL JOURNAL © February 1995

EQUIVALENCE OF REAL SUBMANIFOLDS UNDER VOLUME-PRESERVING HOLOMORPHIC AUTOMORPHISMS OF $\mathbf{C}^n$

FRANC FORSTNERIC

1. Introduction. The main theme of this paper is the global equivalence of certain types of real submanifolds in the complex euclidean space $\mathbf{C}^n$ $(n > 1)$ under the group of all volume-preserving holomorphic automorphisms of $\mathbf{C}^n$. Let Ω be the complex volume form on $\mathbf{C}^n$:

$$\Omega = dz_1 \wedge dz_2 \wedge \cdots \wedge dz_n. \tag{1}$$

A holomorphic mapping $F: D \subset \mathbf{C}^n \to \mathbf{C}^n$ is said to be *volume-preserving* if $F^*\Omega = \Omega$. Since $(F^*\Omega)(z) = JF(z) \cdot \Omega$, where JF is the complex Jacobian of F, this is equivalent to $JF(z) = 1$, $z \in D$. We denote by $\mathrm{Aut}\,\mathbf{C}^n$ the group of all holomorphic automorphisms of $\mathbf{C}^n$ and by $\mathrm{Aut}_1 \mathbf{C}^n \subset \mathrm{Aut}\,\mathbf{C}^n$ the group of all volume-preserving automorphisms of $\mathbf{C}^n$.

Definition 1. Let $\mathscr{G}$ be any group of holomorphic automorphisms of $\mathbf{C}^n$.

(a) Two compact subsets $M_0, M_1 \subset \mathbf{C}^n$ are $\mathscr{G}$-equivalent if there exist a neighborhood U of M_0 in $\mathbf{C}^n$ and a biholomorphic mapping $F: U \to F(U) \subset \mathbf{C}^n$ such that $F(M_0) = M_1$, and F is the uniform limit in U of a sequence $F_j \in \mathscr{G}$.

(b) Let M be a compact topological space. Continuous maps $f_0, f_1: M \to \mathbf{C}^n$ are $\mathscr{G}$-equivalent if there exist a neighborhood U of $f_0(M)$ in $\mathbf{C}^n$ and a biholomorphic mapping $F: U \to F(U) \subset \mathbf{C}^n$ such that $F \circ f_0 = f_1$, and F is the uniform limit in U of a sequence $F_j \in \mathscr{G}$.

Several observations and remarks are in order. For $\mathscr{G} = \mathrm{Aut}\,\mathbf{C}^n$, our Definition 1 agrees with the definition of $\mathbf{C}^n$-equivalence as introduced in [8] (Definition 2). The same definition was used in [7] for the group $\mathrm{Aut}_{\mathrm{sp}}\mathbf{C}^{2n}$ of symplectic holomorphic automorphisms of $\mathbf{C}^{2n}$. If $\mathscr{G} = \mathrm{Aut}_1 \mathbf{C}^n$, it follows that the limit map $F: U \to \mathbf{C}^n$ satisfying Definition 1 is itself volume-preserving. Further, the maximum principle shows that a sequence of holomorphic maps which converges on a neighborhood of a set $K \subset \mathbf{C}^n$ also converges on a neighborhood of the polynomially convex hull $\hat{K}$. Therefore $\mathscr{G}$-equivalence of sets $K_0, K_1 \subset \mathbf{C}^n$ implies $\mathscr{G}$-equivalence of their polynomial hulls. Finally, if $\mathscr{G}' \subset \mathscr{G}$ are holomorphic automorphism groups on $\mathbf{C}^n$ such that $\mathscr{G}'$ is dense in $\mathscr{G}$ (in the topology of uniform

Received 15 April 1994. Revision received 5 September 1994.

Author supported in part by NSF grant DMS-9322766 and by a grant from the Ministry of Science of the Republic of Slovenia.