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ON THE CONVERGENCE OF SPINOR ZETA FUNCTIONS ATTACHED TO HECKE EIGENFORMS ON $Sp_4(\mathbb{Z})$

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Introduction and statement of results. Let F be a nonzero cuspidal Hecke eigenform of even integral weight k on the Siegel modular group $Sp_4(\mathbb{Z})$ of genus 2. We denote by λ_n ($n \in \mathbb{N}$) the Hecke eigenvalues of F. For a prime p we let

$$Z_{F,p}(p^{-s}) := (1 - \lambda_p p^{-s} + (\lambda_p^2 - \lambda_{p^2} - p^{2k-4})p^{-2s} - \lambda_p p^{2k-3} p^{-3s} + p^{4k-6} p^{-4s})^{-1} \qquad (s \in \mathbb{C})$$

be the local spinor zeta function of F at p. We write

$$Z_F(s) := \prod_p Z_{F,p}(p^{-s}) \qquad (\mathrm{Re}(s) \gg 0)$$

for the global spinor zeta function. One has

$$Z_F(s) = \zeta(2s - 2k + 4) \sum_{n \geqq 1} \lambda_n n^{-s} \qquad (\mathrm{Re}(s) \gg 0) \tag{1}$$

(see [1]).

More generally, if χ is a primitive Dirichlet character, we let

$$Z_F(s, \chi) := \prod_p Z_{F,p}(\chi(p)p^{-s}) \qquad (\mathrm{Re}(s) \gg 0)$$

be the twist of $Z_F(s)$ by χ. Thus

$$Z_F(s, \chi) = L(2s - 2k + 4, \chi^2) \sum_{n \geqq 1} \chi(n)\lambda_n n^{-s} \qquad (\mathrm{Re}(s) \gg 0). \tag{2}$$

We are interested in convergence properties of the series on the right of (1) and (2). In [2], Duke, Howe, and Li, using representation-theoretic arguments, showed that

$$\lambda_n = \mathcal{O}_\varepsilon(n^{k-1+\varepsilon}) \qquad (\varepsilon > 0). \tag{3}$$

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