Vol. 76, No. 2 DUKE MATHEMATICAL JOURNAL © November 1994

THE TRACE FORMULA AND DRINFELD'S UPPER HALFPLANE

GERD FALTINGS

1. Introduction. In his groundbreaking paper [Dr] V. G. Drinfeld introduced p-adic symmetric spaces embedded in projective space. They parametrize certain types of formal groups and thus admit coverings defined by division-points of these. He conjectured that the cohomology of these coverings realises in some way the Jacquet-Langlands correspondence between irreducible discrete-series representation of $GL_d(K)$ and irreducible representations of D^*, the multiplicative group of the division algebra D of invariant $1/d$ over the p-adic local field K. This correspondence has been established for $d > 2$ by Deligne and Kahzdan (see [BDKV]).

Carayol has sketched in [C1] a proof for this conjecture for $d = 2$; however, he assumes that the cohomology already decomposes nicely into irreducibles, and especially, that it already constitutes an admissible representation. He uses global methods for this, which also give information on the Galois representations on the cohomology, namely, showing that we also obtain in this way the Langlands correspondence between d-dimensional representations of the Weil group and irreducible representations of $GL_d(K)$.

Here we complete his argument (for $d = 2$) by providing the missing details. Our method is entirely local, but does not give any information on Galois-representations. Its main idea is to apply the Lefschetz trace formula to the (noncompact) symmetric space and to some of its quotients.

We also obtain some information for $d > 2$. However, for higher dimensions the results are less satisfactory for the following reason: the trace formula only gives information for the alternating sum of all cohomology groups. To deal with individual ones one needs vanishing theorems, and these I can show only in the easy case of curves.

As is known, the trace formula in the noncompact case presents some difficulties due to fixed points at infinity. It can be made to work for us precisely because we use it in cases where there should be none of these. That the formula is correct can be shown by using Berkovich's étale cohomology for rigid spaces. However, for curves an elementary proof will be given as an alternative.

Let us mention that P. Schneider and U. Stuhler have investigated the cohomology of these symmetric spaces. They make heavy use of the boundary, which can be done for the symmetric spaces themselves, but not for their coverings. It thus seems that their methods cannot be used here. Nevertheless their work was a

Received 8 November 1993. Revision received 16 May 1994.