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SCATTERING ON A FINITE CHAIN OF VORTICES

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1. Introduction. The vortices mentioned in the title should be interpreted as intersections of the plane with magnetic fluxes concentrated in parallel lines. So the problem is related to the Aharonov-Bohm effect [1]. While the configuration with one vortex enjoys the rotational symmetry and enables an explicit solution by separation of variables (a very brief summary is given in the beginning of Section 5.1), the case of two and more vortices is considerably more complicated. Another feature worth mentioning is that this problem differs somewhat from the usual potential scattering as the Hamiltonian involves only a gauge field and no potential. As explained in [2], instead of the formulation employing a vector potential, the Hamiltonian can be defined by boundary conditions on a cut in the plane. Here, the first coordinate axis serves as the cut since it contains, by our choice, all the vortices.

The aim of this paper is to study this problem in the framework of the mathematical scattering theory. This means to compare quantum time evolution governed by the given Hamiltonian with that governed by the free Hamiltonian. The two Hamiltonians do not differ by a potential but, on the other hand, they are selfadjoint extensions of the same symmetric operator. It is clear that the methods known for potential scattering should be modified or augmented by additional approaches. In view of the relation between the Hamiltonians, Krein's formula suggests itself as a powerful tool, particularly in connection with the Kato-Birman theory. To be able to apply this approach effectively one has to determine the deficiency subspaces $\mathcal{N}(z)$ and to transcribe the boundary conditions into the unitary mapping $V(z)\colon \mathcal{N}(z) \to \mathcal{N}(\bar{z})$ defining the selfadjoint extension.

Evaluating $V(z)$, one is confronted with the following problem. Denote temporarily by $\hat{X}$ and $\hat{P}$ the coordinate and momentum operators in $L^2(\mathbb{R})$, $[\hat{X}, \hat{P}] = i\mathbf{I}$. Let $v\colon \mathbb{R} \to \mathbb{R}$ be a measurable function. The operator

$$\Xi^{-1} = [(\hat{P}^2 + \mathbf{I})^{1/2} + \exp(-2\pi i v(\hat{X}))(\hat{P}^2 + \mathbf{I})^{1/2} \exp(2\pi i v(\hat{X}))]^{-1}$$

is manifestly positive and bounded. If $v \equiv 0$ then $\Xi_0^{-1} = (1/2)(\hat{P}^2 + \mathbf{I})^{-1/2}$. It is desirable to have some information about $\Xi^{-1} - \Xi_0^{-1}$. Perhaps the simplest and most straightforward way to acquire it is to assume that $v \approx 0$ and hence $\exp(2\pi i v(\hat{X}))$ is close to $\mathbf{I}$ and then to attempt a perturbative expansion. Most probably, this naive approach would be possible, provided v is, say, smooth and compactly supported. Unfortunately, this is not the case we are interested in. Our

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