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KOSZUL DUALITY FOR OPERADS

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CONTENTS

0. Introduction

(0.1) The purpose of this paper is to relate two seemingly disparate developments. One is the theory of graph cohomology of Kontsevich [39], [40], which arose out of earlier works of Penner [54] and Kontsevich [38] on the cell decomposition and intersection theory on the moduli spaces of curves. The other is the theory of Koszul duality for quadratic associative algebras, which was introduced by Priddy [55] and has found many applications in homological algebra, algebraic geometry, and representation theory (see, e.g., [5], [6], [7], [30], [51]). The unifying concept here is that of an operad.

This paper can be divided into two parts consisting of Chapters 1, 3 and 2, 4, respectively. The purpose of the first part is to establish a relationship between operads, moduli spaces of stable curves, and graph complexes. To each operad we associate a collection of sheaves on moduli spaces. We introduce, in a natural way, the cobar complex of an operad and show that it is nothing but a (special case of the) graph complex, and that both constructions can be interpreted as the Verdier duality functor on sheaves.

In the second part we introduce a class of operads, called quadratic, and introduce a distinguished subclass of Koszul operads. The main reason for introducing Koszul operads (and in fact for writing this paper) is that most of the operads "arising from nature" are Koszul; cf. (0.8) below. We define a natural duality on quadratic operads (which is analogous to the duality of Priddy [55] for quadratic associative algebras) and show that it is intimately related to the cobar-construction, i.e., to graph complexes.

(0.2) Before going further into discussion of the results of the paper, let us make some comments for the reader not familiar with the notion of an operad. Operads were introduced by J. P. May [52] in 1972 for the needs of homotopy

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