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EVOLUTION OF SEMILINEAR WAVES WITH SWALLOWTAIL SINGULARITIES

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1. Introduction. In this article we consider the formation of caustics for bounded solutions of a semilinear wave equation

$$Pu = f(z, u), \qquad z \in \Omega \subset \mathbb{R}^n, \qquad f \text{ smooth}$$

with P a second order strictly hyperbolic differential operator. We show that in the case of the swallowtail caustic and conormal initial data the only additional singularities of the solution u due to the nonlinearity of the equation lie on the forward characteristic cone over the swallowtail tip.

To illustrate the problem we first consider the formation of caustics for the linear wave equation. Let $\Omega \subset \mathbb{R}^n$, $n \geqslant 3$, be an open subset and (t, x), $x \in \mathbb{R}^{n-1}$, be coordinates in Ω. Let $\Box = D_t^2 - \Delta$, $\Delta = \sum_{j=1}^{n-1} D_{x_j}^2$, be the flat wave operator in Ω. Set

$$\Omega^0 = \Omega \cap \{t = 0\}, \qquad \Omega_{t_0} = \Omega \cap \{t < t_0\}$$

and assume that Ω is a domain of influence of Ω^0, i.e., every maximally extended null bicharacteristic of $\Box$ over Ω intersects Ω^0.

For $s \in \mathbb{R}$ let $u \in H^s_{\text{loc}}(\Omega)$ satisfy

$$\begin{gathered} \Box u = 0, \text{ in } \Omega \\ u = u_0, \qquad D_t u = u_1 \text{ in } \Omega^0, \qquad u_i \in H^{s-i}_{\text{loc}}(\Omega^0), \qquad i = 0, 1. \end{gathered} \tag{1.1}$$

Let $\Sigma^0 = \{g(x) = 0\} \subset \Omega^0$ be a smooth hypersurface and $\Psi^1_P(\Omega^0, \Sigma^0)$ be the space of properly supported pseudodifferential operators of order one in Ω^0 whose principal symbols vanish at $N^*\Sigma^0$. Suppose that the initial data u_0, u_1 are Lagrangian distributions associated to $N^*\Sigma^0$, i.e.,

$$\Psi^1_P(\Omega^0, \Sigma^0)^k u_i \subset H^{s-i}_{\text{loc}}(\Omega^0), \qquad \forall k \in \mathbb{N}. \tag{1.2}$$

Let $\Lambda_0 = N^*\Sigma^0 \subset T^*\Omega$ be the conormal bundle of Σ^0 where Σ^0 is considered as a codimension two submanifold of Ω. For $p = \sigma(\Box) = \tau^2 - |\xi|^2$, the principal

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