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THE CAUCHY PROBLEM FOR HYPERBOLIC OPERATORS OF STRONG TYPE

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1. Introduction. In the seventies, after the success of studies of hyperbolic operators with constant multiple characteristics, there appeared many papers devoted to finding conditions for which the Cauchy problem on hyperbolic operators with smooth, but not constant, multiple characteristics would be C^∞ well posed. It seems that, at least technically, these studies were based on the fact that the principal part of such operators becomes a product of first-order pseudodifferential operators by the smoothness of the characteristics, while the geometry of the characteristics may be so complicated that we rarely get invariant conditions. Many references to papers on the C^∞ hyperbolic Cauchy problem up to the seventies are given in [22].

On the other hand, for second-order hyperbolic operators, the C^∞ Cauchy problem was studied from a more geometrical viewpoint and is now fairly well understood. See [11], [13], [20], [23] for effectively hyperbolic operators and [9], [12] for the noneffectively hyperbolic case. However, see [6] for a complexity of the problem.

Subsequently, influenced by the studies on second-order operators, and hence, from a more geometrical viewpoint than those of the seventies, the C^∞ Cauchy problem on hyperbolic operators with multiple characteristics (not necessarily smooth) were studied. See [18], [24] for operators with characteristics generalizing effectively hyperbolic operators and see [1], [18], [27] for those with characteristics generalizing noneffectively hyperbolic operators.

From the results on second-order operators we know that the C^∞ Cauchy problem is well posed for every lower-order term if and only if the geometry of the characteristics and the localization is simple. So we could expect the existence of hyperbolic operators with characteristics of order greater than two for which the C^∞ Cauchy problem is well posed regardless of *reasonable perturbations* of lower-order terms provided that the geometry of the characteristics and the localization is *simple enough*. To clarify *reasonable perturbation*, we first recall a necessary condition of Ivrii-Petkov [10].

Let P be a differential operator of order m in a bounded open set $\Omega \subset \mathbf{R}^n$, hence a sum of differential polynomials P_j of degree j ($j \leqslant m$) with symbol $P_j(x, \xi)$. The Ivrii-Petkov condition states that if the Cauchy problem for P is well posed in C^∞, then

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