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IRREDUCIBLE MODULAR REPRESENTATIONS OF GL_2 OF A LOCAL FIELD

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Let F be a local field with ring of integers $\mathcal{O}$ and residue field $\mathbb{F}$. We suppose $p = \operatorname{char} \mathbb{F} \neq 0$. The theory of smooth representations of $G = \mathrm{GL}_2(F)$ on complex vector spaces is well understood (we use [BZ] as a general reference). In particular the irreducible objects are classified. Vignéras [Vi] adapted the theory and the classification of irreducibles to representations of G on vector spaces over an algebraically closed field E of characteristic $\ell \neq p$. The case $\ell = p$, in contrast, is very different. This is because G is locally a pro-p-group. Many of the tools one uses when $\ell \neq p$ disappear, others do not work as well. When $\ell = p$ there is no E-valued "Haar measure" on G. More generally, a locally constant E-valued distribution on G is zero, and hence the usual Hecke algebra vanishes. Since F has no nontrivial smooth E-valued characters the theory of Fourier coefficients and Whittaker models collapses. The functors of Γ-invariants ($\Gamma \subset G$ a compact subgroup) and Jacquet restriction cease to be exact, and the theory of contragredient modules and matrix coefficients is very unsatisfactory.

It turns out, nevertheless, that one can give a rough classification of smooth irreducible representations of G along the lines of the unramified case [BL]. Our method is based on defining and studying certain universal objects. Set $K = \mathrm{GL}_2(\mathcal{O})$, let $I \subset K$ be the standard Iwahori subgroup and let $I_1 \subset I$ be the pro-p Sylow subgroup. Let Z be the center of G, and let σ be a smooth irreducible representation of KZ. All such σ's are obtained by inflating an irreducible (finite dimensional) representation σ_0 of $\mathrm{GL}_2(\mathbb{F})$ to K via the reduction map, and extending to KZ by making the center act through scalars. We set $\mathbf{V}_\sigma = \operatorname{ind}_{KZ}^G \sigma$, where ind stands for compactly supported induction. The Hecke algebra $\mathscr{H}_\sigma$ is by definition the algebra $\operatorname{End}_G \mathbf{V}_\sigma$ of intertwining operators. We show that $\mathscr{H}_\sigma \simeq E[T]$ where $T = T_\sigma$ is a specific element, the Hecke operator (Proposition 8). We then show that any smooth irreducible G-module with central character is a quotient of a *standard module* $\mathbf{V}_\sigma/(T - \lambda)\mathbf{V}_\sigma$ for some such σ and $\lambda \in E$ (Theorem 33). Suppose that $\lambda \neq 0$. Then we can make a complete analysis of the standard modules. We show that they are usually irreducible. They always have length at most 2 and a unique irreducible quotient, isomorphic to a principal series representation, a special representation or a one-dimensional representation (see Proposition 29 and Theorems 30 and 33). We determine all isomorphisms among them (Corollary 36). Our result is actually more precise in that we give embeddings of the localization $\mathbf{V}_\sigma[T_\sigma^{-1}]$ into "universal" principal series, viewing both as alge-

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