Vol. 74, No. 2 DUKE MATHEMATICAL JOURNAL © May 1994

ON THE TOPOLOGY OF NONNEGATIVELY CURVED SIMPLY CONNECTED 4-MANIFOLDS WITH DISCRETE SYMMETRY

DAGANG YANG

1. Introduction. A fascinating problem in Riemannian geometry is the study of the topology of positively curved compact 4-manifolds. The first interesting result is probably due to A. Lichnerowicz in [14], where he proved that a $4k$-dimensional compact spin manifold with nonzero $\hat{A}$-genus carries no metric with positive scalar curvature. In dimension 4, the $\hat{A}$-genus reduces to a constant multiple of the signature. It follows from the topological classification of smooth simply connected compact 4-manifolds by S. K. Donaldson [4] and M. H. Freedman [6] that the only smooth simply connected compact spin 4-manifolds that can carry metrics of positive scalar curvature are homeomorphic to the connected sums of S^4 with finitely many copies of $S^2 \times S^2$. Namely,

$$(*) \qquad S^4, \qquad \#_{i=1}^{l} S^2 \times S^2, \qquad l = 1, 2, 3, \ldots .$$

On the other hand, a smooth simply connected compact nonspin 4-manifold is homeomorphic to one of the following manifolds:

$$(**) \qquad \#_{j=1}^{k} \pm \mathbf{CP}^2, \qquad \mathbf{k} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \ldots .$$

The results in [8] by M. Gromov and H. B. Lawson imply that both the manifolds listed in (∗) and (∗∗) carry metrics with positive scalar curvature. For positive Ricci curvature, it is shown by J. P. Sha and the author in [18], [19] that, for smooth simply connected compact 4-manifolds, there is no other topological obstructions besides the $\hat{A}$-genus. More specifically, each 4-manifold listed in (∗) and (∗∗) carry smooth metrics with positive Ricci curvature. Thus the topological classification problem for compact simply connected 4-manifolds with positive scalar curvature or positive Ricci curvature has been solved. On the other hand, the only known examples of compact simply connected 4-manifolds with positive sectional curvature (henceforth abbreviated to positive curvature) are S^4 and $\mathbf{CP}^2$. Both of them are homogeneous manifolds. The only known examples of compact simply connected 4-manifolds with nonnegative sectional curvature are S^4, $\mathbf{CP}^2$, $S^2 \times S^2$, and $\pm \mathbf{CP}^2 \# \mathbf{CP}^2$. The first three manifolds are homogeneous, but the last two manifolds are nonhomogeneous, and they were constructed by J. Cheeger in [2]. With additional conditions, it is shown by M. Berger [1] that, for a 4-dimensional Einstein

Received 22 June 1993. Revision received 1 November 1993.
Author's research partially supported by NSF grant DMS 90-03524 and 9209330.