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A VARIATIONAL MIXED TORELLI THEOREM

KĘSTUTIS IVINSKIS

The Torelli problem for a family of varieties deals with the question of whether the period map from the base space of this family to a period matrix space is injective. Strong Torelli-type theorems have been proved using the infinitesimal variation of Hodge structure (IVHS) techniques of Carlson et al. in [CG] and [CGGH]. The idea is to prove a generic Torelli theorem which states that the period map for the varieties in question has degree 1 onto its image. A result of Cox, Donagi, and Tu [CDT] allows us to reduce it to the variational Torelli problem. In a Torelli problem, one seeks to recover a variety from the algebraic data of its period map, but in a variational Torelli problem, the algebraic data comprise not only the period map but also its derivative.

By adapting Donagi's symmetrizer technique [Do] as used by Green [Gre] and by applying a new way of recovering the variety in question from its IVHS, we show in this article a variational mixed Torelli theorem.

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1. Discussion of the main result. Let Y be a complex, smooth, and projective variety of dimension n and $\mathscr{L}$ an invertible ample sheaf on Y. We recall the notion of a sufficiently ample invertible sheaf.

Definition 1.1 [Gre]. A property holds for *sufficiently ample* invertible sheaves $\mathscr{F}$ on Y if there exists an ample invertible sheaf $\mathscr{F}_0$ on Y so that the property holds for all $\mathscr{F}$ on Y with $\mathscr{F} \otimes \mathscr{F}_0^{-1}$ ample. We denote this by saying the property holds for $\mathscr{F} \gg 0$.

Let X be a smooth, reduced divisor on Y with $\mathscr{L} = \mathscr{O}_Y(X)$. By Usui's [Us] transfer of Griffiths's work on period maps to the case of variation of mixed Hodge structures arising from families of logarithmic deformations, the cohomological interpretation of the infinitesimal mixed period map is given in this case by the map δ_Y^{log}

$$H^1(Y, T_Y\langle -X\rangle) \xrightarrow{\delta_Y^{log}} \bigoplus_{p=1}^{n} \mathrm{HOM}_{\mathbb{C}}(H^{n-p}(Y, \Omega_Y^p\langle X\rangle), H^{n-p+1}(Y, \Omega_Y^{p-1}\langle X\rangle)).$$

We write $T_Y\langle -X\rangle$ for the more commonly accepted $T_Y\langle -log\ X\rangle$. Griffiths [Gri] shows that the map δ_Y^{log} is injective assuming $\mathscr{L} \gg 0$ (compare to [SSU, §7]). This

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