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THE WHITHAM-TYPE EQUATIONS AND LINEAR OVERDETERMINED SYSTEMS OF EULER-POISSON-DARBOUX TYPE

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1. Introduction. It is known that the zero-dispersion limit of the KdV equation can be described by the Whitham-type equations [8], [14]

$$\frac{\partial \beta_i}{\partial t} + \lambda_i(\beta_1, \beta_2, \ldots, \beta_{2g+1})\frac{\partial \beta_i}{\partial x} = 0, \qquad i = 1, 2, \ldots, 2g+1. \tag{1.1}$$

with the ordering

$$\beta_{2g+1} < \beta_{2g} < \cdots < \beta_1$$

where λ_i's, which are given later, depend on complete hyperelliptic integrals of genus g. Equations (1.1) were also found by Whitham [15] in the single-phase case $g = 1$, and more generally by Flaschka, Forest, and McLaughlin [3] in the multiphase cases.

The structure of the Whitham-type equations (1.1) has been a subject of recent studies. Their hyperbolic nature was shown by Levermore [9]. Novikov and Dubrovin [1], [2] found the geometric-Hamiltonian structure for equations (1.1). Based on this Hamiltonian structure, Tsarev [13] made a remarkable observation that equations (1.1) can be solved by a hodograph transform. This was put into an algebro-geometric setting by Krichever [4].

Tsarev's hodograph method enabled the author [10], [11] to further transform equations (1.1) in the single-phase case into a linear overdetermined system of Euler-Poisson-Darboux type

$$2(\beta_i - \beta_j)\frac{\partial^2 q}{\partial \beta_i \partial \beta_j} = \frac{\partial q}{\partial \beta_i} - \frac{\partial q}{\partial \beta_j} \qquad i, j = 1, 2, 3 \tag{1.2}$$

$$q(\beta, \beta, \beta) = F(\beta) \qquad i \neq j.$$

System (1.2) has a unique solution, and the solution can be written explicitly. Using the explicit formula for solutions to system (1.2), the author was able to solve an initial value problem for the single-phase Whitham-type equatons. This initial value problem is important in Lax-Levermore-Venakides theory [8], [14] to determine the KdV zero-dispersion limit.

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