Vol. 74, No. 1 DUKE MATHEMATICAL JOURNAL © April 1994

NORMAL FORMS OF REAL SURFACES UNDER UNIMODULAR TRANSFORMATIONS NEAR ELLIPTIC COMPLEX TANGENTS

XIANGHONG GONG

1. Introduction. Let $M \subset \mathbb{C}^2$ be a real analytic surface with a nondegenerate complex tangent at p. Then for a suitable choice of unimodular coordinates, one may assume that $p = 0$ and

$$(1.1) \qquad M\colon z_2 = z_1\bar{z}_1 + \gamma z_1^2 + \bar{\gamma}\bar{z}_1^2 + q(z_1, \bar{z}_1), \qquad \gamma \in \mathbb{C},$$

in which $q(z_1, \bar{z}_1)$ is a convergent power series in z_1 and $\bar{z}_1$ starting with the terms of the third order. The γ in (1.1) is a unimodular invariant and $|\gamma|$ is the Bishop invariant [1]. The complex tangent is said to be *elliptic*, *parabolic*, or *hyperbolic* according to $0 \leqslant |\gamma| < 1/2$, $|\gamma| = 1/2$ or $1/2 < |\gamma| < \infty$. In [4], J. K. Moser and S. M. Webster investigated systematically the normal forms of real analytic surfaces in the form (1.1) under the biholomorphic change of coordinates. The problem of normal forms of real analytic surfaces under holomorphic unimodular transformations was further studied in [2], where the Moser-Webster normal form played an important role. In particular, it was shown that a real analytic surface near an elliptic complex tangent with nonvanishing Bishop invariant can be transformed into a normal form under holomorphic unimodular transformations.

In this paper, we discuss the normal form of surfaces near an elliptic complex tangent with $\gamma = 0$.

THEOREM 1.1. *Let M be a real analytic surface of elliptic complex tangent at 0 with $\gamma = 0$. Then there exists a unique formal unimodular mapping which transforms M into*

$$(1.2) \qquad \tilde{M}\colon z_2 = z_1\bar{z}_1 + 2\,\mathrm{Re}\{z_1^2 a(z_1, z_1\bar{z}_1)\},$$

in which $a(z_1, z_1\bar{z}_1)$ is a formal power series in z_1 and $\bar{z}_1$ without the constant term. Moreover, if $a(z_1, z_1\bar{z}_1) = 0$, then the transformation converges.

The approach presented here is different from [2], since for the case $\gamma = 0$ the problem of normal forms under biholomorphic transformations has not been fully understood (see [4], [3]). The difficulty involved in the case $\gamma = 0$ is that the intrinsic characterization of a complex tangent by a pair of involutions given in [4] exists only for $\gamma \neq 0$. Furthermore, the automorphism of quadric $Q_0\colon z_2 = z_1\bar{z}_1$ is quite

Received 8 June 1993.