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DISTRIBUTION OF ENERGY LEVELS OF A QUANTUM FREE PARTICLE ON A SURFACE OF REVOLUTION

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1. Introduction. Let M be a two-dimensional smooth compact manifold which is homeomorphic to a sphere, and which is a surface of revolution in $\mathbf{R}^3$, with an axis A and poles N and S (see Figure 1). The geodesic flow on M is a classical integrable system due to the Clairaut integral,

$$r \sin \alpha = \text{const}. \tag{1.1}$$

In the present work we are interested in high energy levels of the corresponding quantum system,

$$-\Delta u_n = E_n u_n. \tag{1.2}$$

Let s be the normal coordinate (the length of geodesic) along the meridian, and let

$$r = f(s), \qquad 0 \leqslant s \leqslant L, \tag{1.3}$$

be the equation of M, where r is the radial coordinate. Then

$$\Delta = f(s)^{-1} \frac{\partial}{\partial s}\left(f(s) \frac{\partial}{\partial s} \right) + f(s)^{-2} \frac{\partial^2}{\partial \varphi^2}, \tag{1.4}$$

where φ is the angular coordinate.

We assume that $f(s)$ has a simple structure, so that

$$f'(s) \neq 0,\ s \neq s_{\max}; \qquad f''(s_{\max}) \neq 0, \tag{1.5}$$

where

$$f(s_{\max}) = \max_{0 \leqslant s \leqslant L} f(s) \equiv f_{\max}.$$

For normalization we put $f_{\max} = 1$. Another assumption on M is the following twist hypothesis.

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