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APPENDIX: AN EXOTIC FOURIER TRANSFORM FOR H_4

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0. Introduction. In [5], Lusztig gives the definition of a *fusion datum.* Such fusion data naturally occur in connection with two-sided cells in finite Coxeter groups. In that situation, they are called Fourier transforms. The Fourier transforms for all cells for all finite Weyl groups are determined by Lusztig in [3]. For the cells in H_3 and the small cells in H_4, the matrix is easy to find and is given in [2]. For the dihedral groups $I_2(p)$, Lusztig determines a Fourier transform matrix in [5]. Here we complete the picture by giving a (74×74)-matrix connected to the big two-sided cell in the Weyl group of type H_4, which is the last missing case.

1. Fusion data. For the convenience of the reader we recall the properties of a certain type of fusion datum from [5]. Let $S = (s_{k,l})_{k,l} \in \mathbb{R}^{n \times n}$ be a real symmetric matrix, and let $\mathbf{t} = (t_k) \in \mathbb{C}^n$ be a vector of complex roots of unity. Further, let π be an involutory permutation of $\{1, \ldots, n\}$ with a fixed point k_0, $1 \leqslant k_0 \leqslant n$. If

(1) $s_{k,l} = s_{\pi(k),\pi(l)}$ for all $1 \leqslant k, l \leqslant n$,
(2) $t_k = \bar{t}_{\pi(k)}$ for all $1 \leqslant k \leqslant n$,
(3) $\sum_{j=1}^{n} s_{k,j} s_{j,l} = \delta_{k,l}$ for all $1 \leqslant k, l \leqslant n$,
(4) $s_{k,k_0} > 0$ for all $1 \leqslant k \leqslant n$,
(5) $t_{k_0} = 1$,
(6) $\sum_{j=1}^{n} \dfrac{s_{k,j} s_{l,j} s_{m,j}}{s_{k_0,j}} \in \mathbb{N}$ for all $1 \leqslant k, l, m \leqslant n$, and
(7) $\sum_{j=1}^{n} t_j s_{j,k} s_{l,j} = t_k t_l s_{\pi(k),l}$ for all $1 \leqslant k, l \leqslant n$,

then by the definition in [5] the tuple $(\{1, \ldots, n\}, k_0, \pi, \pi, S, \mathbf{t})$ is a fusion datum.

The interest in such fusion data stems from the fact that the base change matrix between unipotent character degrees and the degrees of fake characters for families of unipotent characters in finite groups of Lie type are examples of matrices S of the above type, with the vector $\mathbf{t}$ given by the eigenvalues of Frobenius attached to the occurring unipotent characters (see [5]). In this case the matrix S is called a nonabelian Fourier transform.

The Fourier transforms for all families of unipotent characters for all finite groups of Lie type are determined by Lusztig in [3]. There it is also shown that every such matrix is connected to a finite group Γ, either an elementary abelian 2-group or one of the symmetric groups S_3, S_4 or S_5, in such a way that the rows and columns of S may be indexed by pairs (σ, χ), where σ is an element of Γ and χ an irreducible

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