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MAPPING PROPERTIES OF THE BERGMAN PROJECTION ON CONVEX DOMAINS OF FINITE TYPE

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Introduction. The purpose of this paper is to prove sharp Sobolev space and Lipschitz space estimates for the Bergman projection operator associated to a smoothly bounded, convex domain of finite type, Ω, in $\mathbb{C}^n$. Our estimates are of two types: isotropic ones, i.e., for the usual L_k^p Sobolev spaces and the classical Λ_α Lipschitz spaces, and nonisotropic ones, for certain Γ_α spaces defined in terms of the geometry of the boundary of Ω.

This article is the first of two papers, the second dealing with the somewhat more intricate analogues for the Szegö projection operator. Common to both papers are certain geometric constructions and related kernel estimates obtained by one of us in [Mc1]; in addition, techniques developed in [NRSW] and [CNS] will also play a role. We begin by describing, in somewhat imprecise terms, elaborations and reformulations of some of the ideas in [Mc1] which will be crucial to what follows.

There are, to begin with, three related geometric objects:

(i) for each $p \in b\Omega$ and $\varepsilon > 0$, the polydisc $P_\varepsilon(p)$ which is roughly speaking the largest polydisc centered at p contained in an ε-outward "translate" of Ω; more generally for $p \in \Omega$, $P_\varepsilon(p)$ is the polydisc centered at p just reaching out to the hypersurface $\{z: r(z) = \varepsilon + r(p)\}$, where r is the defining function of Ω;

(ii) the family of balls $B(p, \varepsilon) = P_\varepsilon(p) \cap b\Omega$ on the boundary, for each $p \in b\Omega$; this construct makes $b\Omega$ a "space of homogeneous type", by virtue of the fact that the balls $B(p, \varepsilon)$ satisfy both the Vitali engulfing property and the doubling property with respect to dilation of the "radii" ε; and

(iii) the family of "tents" $T(p, \varepsilon) = P_\varepsilon(p) \cap \Omega$ for $p \in b\Omega$, whose bases are the balls in (ii).

Connected with these geometric notions are the metric quantities:

(a) a quasi distance M, defined in Ω, with the property that $M(p, q) \approx \inf\{\varepsilon > 0: q \in P_\varepsilon(p)\}$;

(b) a quasi distance ρ, defined on $b\Omega$, with the property that $\rho(p, q) \approx \inf\{\varepsilon > 0: q \in B(p, \varepsilon)\}$;

(c) the "natural" extension of ρ to $\overline{\Omega}$, with

$$\rho(p, q) \approx \min\{|p - q|, \inf\{\varepsilon: q \in B(\pi(p), \varepsilon)\}\},$$

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