Vol. 72, No. 3 DUKE MATHEMATICAL JOURNAL © December 1993

SYMPLECTIC EMBEDDING TREES FOR GENERALIZED CAMEL SPACES

LISA TRAYNOR

1. Overview. A basic problem in symplectic geometry is to understand when two symplectic manifolds are equivalent. The *symplectic mapping problem* is this equivalence question addressed to open subsets of the standard symplectic space $(\mathbb{R}^{2n}, \omega_0 = dx_1 \wedge dy_1 + \cdots + dx_n \wedge dy_n)$: *Given two open, diffeomorphic subsets U, V of $\mathbb{R}^{2n}$, when does there exist a diffeomorphism $\psi\colon U \to V$ such that $\psi^*\omega_0 = \omega_0$?*

Although no general classification scheme is known, in recent years progress has been made on this problem. Via the theory of capacities and, more recently, symplectic homology, basic open and bounded shapes such as ellipsoids and polydiscs can be symplectically classified. (See, for example, [EkH1], [EkH2], [H], [FH], [FHW].) In the following, holomorphic techniques ([G], [E]) are applied to study the mapping problem for particular open, infinite volume subsets of $\mathbb{R}^4$.

As a starting point for the types of spaces examined, consider $C(\lambda) \subset \mathbb{R}^4$, which is defined as

$$C(\lambda) = \{y_1 < 0\} \cup \{y_1 > 0\} \cup H(\lambda)$$

where

$$H(\lambda) = \{x_1^2 + x_2^2 + y_2^2 < \lambda^2, y_1 = 0\}.$$

In [MT], the following theorem was proven.

THEOREM 1.1. *$C(\lambda)$ is symplectically equivalent to $C(\mu)$ if and only if $\lambda = \mu$.*

This result makes these spaces seem very delicate with respect to the "shape" of the hole. In fact, it is possible to change the shape of $H(\lambda)$ without producing a symplectically different space. See Theorem 4.2.2 for a precise criterion of alternate hole shapes. In particular, the following result holds:

THEOREM 1.2. *If $G(\lambda)$ is one of the following sets*
(1) $\{(x_1/a)^2 + (x_2/b)^2 + (by_2)^2 < \lambda^2, y_1 = 0, a, b > 0\}$ *(ellipsoid),*
(2) $\{x_2^2 + y_2^2 < \lambda^2, |x_1| < 1, y_1 = 0\}$ *(bounded cylinder),*
(3) $\{x_2^2 + y_2^2 < \lambda^2, y_1 = 0\}$ *(infinite cylinder),*
then $C'(\lambda) := \{y_1 < 0\} \cup \{y_1 > 0\} \cup G(\lambda)$ is symplectically equivalent to $C(\lambda)$.

Received 11 March 1993.
Research at SUNY Stony Brook supported in part by an Alfred P. Sloan Doctoral Dissertation Fellowship. Research at MSRI supported in part by NSF grant #DMS 9022140.