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ON HIGHER-ORDER DIFFERENTIALS OF THE PERIOD MAP

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Given a variation of Hodge structure (VHS) parametrized by a complex manifold S, we have the corresponding period map Φ from S to the appropriate classifying space of Hodge structures (= the period domain). The map Φ is holomorphic.

Griffiths analyzed the (first) differential of Φ, which turned out to possess a rich structure later codified in the notion of the infinitesimal variation of Hodge structure (see [CGGH]). In [G1] Griffiths also explained how to compute it when the VHS comes from a family of compact Kähler manifolds: $d\Phi$ is given by cup product with the Kodaira-Spencer class of the family. We wish to do the same for higher-order differentials of Φ—explain what they are and how to compute them for the VHS that comes from geometry.

We start by replacing Hodge structure with an infinite-dimensional analogue, Archimedean cohomology (used by Deninger [Den] to understand the Γ-factors at ∞). This results in some simplifications: instead of a varying flag of subspaces of a (finite-dimensional) vector space, we work with a single subspace moving in an (infinite-dimensional) vector space; this also provides enough room to separate various higher-order differentials and their components. We note that this approach is somewhat analogous to M. Saito's construction of the "period map via Brieskorn lattices" for an unfolding of a holomorphic function with an isolated critical point [S].

Our main result (Theorem 3 in Section 5.4) is a recipe for computing the higher-order differentials of the "Archimedean period map," from which one can also obtain the differentials of the usual period map.

Roughly speaking, for the period map arising from a deformation of a variety X, the differentials of order k are induced on the Archimedean cohomology of X by a kind of cup product with certain expressions Π_α constructed from the data (up to kth order) of the Kodaira-Spencer mapping of the deformation. In order to define these Π_α's, we introduce some cochain operations. The appearance of explicit cochains is unavoidable, since a higher-order deformation cannot be described in conventional cohomological terms; however, in Theorem 4 of Section 5.7, we prove that the construction is independent of the choice of the Čech cochains representing the Kodaira-Spencer mapping.

There are a few places in the literature where the higher-order differentials of the period map appear in some form. In [CGGH] and [G2] there is an extended

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