Vol. 72, No. 1 DUKE MATHEMATICAL JOURNAL © October 1993

TRANSCENDENTAL CYCLES ON ORDINARY K3 SURFACES OVER FINITE FIELDS

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1. Introduction. Let Z be a complex algebraic K3 surface, and $V(Z)$ the $\mathbb{Q}$-lattice of transcendental cycles on Z. By definition, $V(Z)$ is the orthogonal complement of $\mathrm{NS}(Z) \otimes \mathbb{Q}$ of the second rational cohomology group $H^2(Z, \mathbb{Q})$ with respect to the intersection pairing. Here $\mathrm{NS}(Z)$ is the Neron-Severi group of Z. It is well known that $V(Z)$ carries a natural rational Hodge structure of weight 2. In [28] we have proven that this structure is *irreducible* and its endomorphism algebra is a number field.

Now let Y be an ordinary K3 surface over a finite field k of characteristic p. We write Y_a for $Y \times k(a)$ where $k(a)$ is an algebraic closure of k. For each rational prime l different from p, let us consider the second twisted l-adic cohomology group $H^2(Y_a, \mathbb{Q}_l)(1)$ of Y_a. The Galois group $G(k)$ of k acts on $H^2(Y_a, \mathbb{Q}_l)(1)$ in a natural way. One may identify $\mathrm{NS}(Y_a)_l = \mathrm{NS}(Y_a) \otimes \mathbb{Q}_l$ with a certain Galois-invariant subspace of $H^2(Y_a, \mathbb{Q}_l)(1)$, and a theorem of Nygaard [12] asserts that this subspace coincides with $G(k)$-invariants $H^2(Y_a, \mathbb{Q}_l)(1)^{G(k)}$ if k is "sufficiently large". (This theorem proves a special case of a general conjecture due to Tate [19].) We define the $\mathbb{Q}_l$-lattice $V_l(Y)$ as the orthogonal complement of $\mathrm{NS}(Y_a)_l$ in $H^2(Y_a, \mathbb{Q}_l)(1)$ with respect to the intersection pairing. Recall that this pairing and its restriction to $\mathrm{NS}(Y_a)_l$ are nondegenerate. This gives us a canonical splitting

$$H^2(Y_a, \mathbb{Q}_l)(1) = \mathrm{NS}(Y_a)_l \oplus V_l(Y).$$

Since the intersection pairing is Galois-invariant, $V_l(Y)$ is a Galois-invariant subspace and the splitting above is also Galois-invariant. Recall that $G(k)$ is procyclic and has a canonical generator, namely, the arithmetic Frobenius automorphism

$$\sigma_k\colon k(a) \to k(a), \qquad x \to x^q$$

where q is the number of elements of k. Clearly, q is an integral power of p. Another canonical generator of $G(k)$ is the geometric Frobenius automorphism $\varphi_k = \sigma_k^{-1}$.

In this paper we examine the characteristic polynomial

$$P_{Y,\mathrm{tr}}(t) := \det(id - t\varphi_k, V_l(Y)).$$

Received 28 December 1992.
Author supported by the Netherlands Organization for Scientific Research (N.W.O.).