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AUTOMORPHISMS AND THE KÄHLER CONE OF CERTAIN CALABI-YAU MANIFOLDS

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Let X be the fiber product over $\mathbb{P}^1$ of two rational elliptic surfaces with section as in the diagram

$$\begin{tikzcd} & X \arrow[dl, "p_1"'] \arrow[dr, "p_2"] & \\ S_1 \arrow[dr, "\pi_1"'] & & S_2 \arrow[dl, "\pi_2"] \\ & \mathbb{P}^1 & \end{tikzcd}$$

and let $\varphi = \pi_1 \cdot p_1 = \pi_2 \cdot p_2 \colon X \to \mathbb{P}^1$. Schoen [S] has shown that, if the surfaces are sufficiently general (see §1 and §3 below for the precise condition), then X is a smooth Calabi-Yau manifold. Let $\mathscr{K}(X)$ be the Kähler cone of X and let $\overline{\mathscr{K}(X)}$ be its closure. Since $h^{2,0}(X) = 0$, the Kähler cone is the convex hull of the set $\mathscr{K}(X) \cap H^2(X, \mathbb{Q})$ of ample $\mathbb{Q}$-divisor classes on X. We define the *nef cone* $\mathscr{K}(X)_+$ to be the convex hull of the set $\overline{\mathscr{K}(X)} \cap H^2(X, \mathbb{Q})$ of nef $\mathbb{Q}$-divisor classes. (This cone consists of the Kähler cone $\mathscr{K}(X)$ together with that part of the boundary of its closure which is rationally defined.) For a fiber product of rational elliptic surfaces with section, the nef cone is known to have infinitely many edges. Here we show that there is a fundamental domain which is a (finite) rational polyhedral cone, for the induced action of $\mathrm{Aut}(X)$ on $\mathscr{K}(X)_+$.

Our work was inspired by some recent conjectures of the second author [M1, M2] which derive from the "mirror symmetry" phenomenon for Calabi-Yau manifolds. In [M1], some of the data from the topological field theories introduced by Witten [W1, W2] is used to construct some novel variations of Hodge structure from Calabi-Yau manifolds. In [M2], the implications of this construction for possible compactifications of moduli spaces are explored. In particular, it is pointed out there that Looijenga's semitoric compactification method [L2] can be fruitfully applied in this situation provided that the action of the fundamental group on the nef cone has a rational polyhedral fundamental domain. This paper provides the first nontrivial example of such a structure.

General results of Wilson [Wi] tell us that away from its intersection with the cubic cone W^* defined by cup-product, the closure $\overline{\mathscr{K}}$ of the Kähler cone of a

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