Vol. 71, No. 2 DUKE MATHEMATICAL JOURNAL © August 1993

TOPOLOGICAL INVARIANCE OF THE SOLVABILITY OF COMPLEX-VALUED VECTOR FIELDS IN THE PLANE

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0. Introduction. We know from [8] that linear partial differential equations with complex-valued coefficients may not always be solvable. In particular, in $\mathbb{R}^2$, for a vector field

$$L = \frac{\partial}{\partial y} - \sqrt{-1}b(x, y)\frac{\partial}{\partial x}, \qquad \text{with } b(x, y) \in C^\infty,$$

the equation $Lu = f$ is always solvable if and only if L satisfies condition P, that is, if for every fixed x the function $b(x, \cdot)$ does not change sign (see [12] for systems in higher dimensions). In this paper, we consider a germ at $0 \in \mathbb{R}^2$ of a real-analytic function z, with $dz(0) \neq 0$, and estimate the size of the obstruction to the solvability, for $u \in C^\infty$, of the equation

$$Lu = f, \qquad \text{with} \quad f \in C^\infty \text{ and } L = z_x\frac{\partial}{\partial y} - z_y\frac{\partial}{\partial x},$$

in terms of the cohomology group H^1_{dz}: cohomology relative to dz (see Section 1 for definitions), and then we show that this cohomology group is a topological invariant. More precisely, if L and L' are topologically equivalent vector fields at $0 \in \mathbb{R}^2$, then the corresponding cohomology groups are the same.

The organization of this paper is as follows: in Section 1 we set the terminology and state the main results; in Sections 2, 3, and 4 we prove the main results; and in Section 5 we classify the structures that are topologically equivalent to the Mizohata structure.

1. Terminology and statement of main results. In this section we set notation and terminology, and state the main results of this paper.

We will denote by $\mathcal{O}^n$, $\mathcal{E}^n$, and $\mathcal{F}^n$ the spaces of complex-valued germs at $0 \in \mathbb{R}^n$ of, respectively, real-analytic, smooth (C^∞) functions, and flat functions at 0 (a function is *flat* on a subset $S \subset \mathbb{R}^n$ if it vanishes on S together with all its partial derivatives). More generally, if $A \subset \mathbb{R}^n$, $A \ni 0$, is a real-analytic variety, then we denote by $\mathcal{O}^n(A)$ the space of germs along A of real-analytic functions in $\mathbb{R}^n$; by $\mathcal{E}^n(A)$ the space of germs along A of C^∞ in $\mathbb{R}^n$; by $\mathcal{F}^n(A)$ the space of germs along

Received 29 July 1992.