

ASYMPTOTIC COMPLETENESS FOR $N \leq 4$ PARTICLE
SYSTEMS WITH THE COULOMB-TYPE INTERACTIONS

I. M. SIGAL AND A. SOFFER

CONTENTS

1. Introduction	243
2. Hamiltonians and kinematics	247
3. Asymptotic completeness	249
4. Asymptotic energy cutoffs and sharp energy localization	251
5. Proof of asymptotic completeness	254
6. Time-dependent observables	260
7. Propagation estimates revisited	265
8. Asymptotic clustering revisited	273
9. Many scales of configuration space	275
10. Localization of momentum	278
11. Evolution of threshold channels	282
12. Estimates for free evolution	286
13. Subballistic estimates	290
Appendix	295
Supplement	296
References	297

1. Introduction. In this paper we study the scattering theory for many-body long-range systems. It was known since the foundation of quantum mechanics that the scattering theory for long-range systems is different from that for the short-range ones. However, only when one addresses the problem of many-body asymptotic completeness does one realize the extent of this difference. We discuss the latter within the framework of phase-space analysis, which is the only approach which so far has given access to the long-range many-body problem.

There are two new problems one faces in passing from short-range systems to long-range ones. First, one has to prove sharper propagation estimates, namely, to show that in a certain sense there is no propagation outside a parabolic conical neighbourhood of the subset of the extended phase-space (i.e. including the time and energy axes) determined by the classical trajectories of quantum-mechanically

Received 7 December 1991. Revision received 26 June 1992.

Sigal is an I. K. Killam Research Fellow.

Sigal supported by NSERC under Grant NA7901 and by NSF under Grant DMS-8808032.

Soffer is an A. Sloan Fellow in Mathematics.

Soffer supported by NSF under Grant DMS85-07040.