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AN INVERSE BOUNDARY VALUE PROBLEM IN ELECTRODYNAMICS

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1. Introduction and statement of results. Consider the following inverse boundary value problem of electrodynamics: Let Ω be a bounded body in $\mathbb{R}^3$ whose electric permittivity, conductivity, and magnetic permeability are described by the functions ε, σ, and μ, respectively. The objective is to reconstruct these parameters in a noninvasive way from electromagnetic field measurements on the surface of the body. To be more specific, let E and H denote any electric and magnetic fields inside the body with a harmonic time dependence. Thus, E and H satisfy Maxwell's equations

$$\nabla \wedge E = i\omega\mu H, \quad \nabla \wedge H = -i\omega\left(\varepsilon + i\frac{\sigma}{\omega}\right)E \qquad \text{in } \Omega \tag{1.1}$$

with $\omega > 0$ fixed. Let n be the exterior unit normal on the boundary Γ of Ω. By Λ we denote the linear mapping that assigns the tangential component of $E|_\Gamma$ to that of $H|_\Gamma$, i.e.,

$$\Lambda(n \wedge E) = n \wedge H.$$

The question that is addressed in the present article is: Can one recover uniquely the functions ε, σ, and μ from the knowledge of the mapping Λ?

The roots of this problem are in the inverse boundary value problem of electrostatics. In his article [C], Calderón asked if the conductivity of the body is uniquely determined by the voltage-to-current (or Dirichlet-to-Neumann) map on the surface of the body. This problem obtained considerable attention by a number of authors, and the problem of uniqueness was solved affirmatively in space dimensions greater than two (see [HN], [KV], [I], [N], [R], and [SU]) and relatively generally also in two space dimensions ([SuU1]). One of the crucial tools in tackling this problem turned out to be related to an old method due L. D. Faddeev who, in a different connection, constructed free-space fundamental solutions with an exponential growth in certain directions. (See also [Ne] for a detailed discussion of this fundamental solution.) An application of Green's formula with this fundamental solution transforms the problem into an asymptotically linear one (see [SU] for details). A further step was taken in the article of Nachman [N], where it was

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