Vol. 69, No. 2 DUKE MATHEMATICAL JOURNAL © February 1993

ON THE NONVANISHING OF RANKIN-SELBERG L-FUNCTIONS

WENZHI LUO

1. Introduction. The existence and construction of cusp forms for Fuchsian groups of the first kind has been a difficult problem for many years, defying all kinds of attack. In his 1954 Göttingen lectures on the theory of Eisenstein series, Selberg [10] expressed the view that the space of discrete spectrum should always be infinite-dimensional, and moreover that the discrete spectrum should dominate over the continuous one in a certain quantitative sense. But recently, to everybody's surprise, R. S. Phillips and P. Sarnak [9] have revealed a very different truth, that cusp forms are rare and rather fragile objects whose existence may well be limited to certain arithmetic groups. In [9], by considering the Teichmüller space for $\Gamma_0(N)$, they examined the behavior of a Maass cusp form $u_j(z)$ for $\Gamma_0(N)$ under quasi-conformal deformations determined by a holomorphic cusp form $Q(z)$ of weight 4. They established a sufficient condition for annihilating the cusp form, namely, that the corresponding Rankin-Selberg L-function $L(Q \otimes u_j, s)$ does not vanish at the special point $s_j = \frac{1}{2} + it_j$ with $\lambda_j = s_j(1 - s_j)$ being the eigenvalue for $u_j(z)$.

Motivated by the above research, J.-M. Deshouillers and H. Iwaniec [1] proved a spectral square mean-value theorem for these L-functions, which implied that

$$\#\{j;\, t_j \leqslant T,\, L(Q \otimes u_j, s_j) \neq 0\} \gg T^{1-\varepsilon}.$$

See also [2].

In this work we achieve an essentially best-possible nonvanishing result by establishing a new spectral mean-value theorem and appealing to the recent remarkable results of Iwaniec ([7]) and Hoffstein and Lockhart ([6]) on the order of magnitude of the first Fourier coefficients of Maass cusp forms. We prove that

$$\#\{j;\, t_j \leqslant T,\, L(Q \otimes u_j, s_j) \neq 0\} \gg T^{2-\varepsilon}.$$

Hence, assuming that the multiplicity of λ_j is bounded by t_j^{ε}, we conclude from the Phillips-Sarnak theory that, for generic $\Gamma \subseteq SL(2, \mathbf{R})$, the determinant of the scattering matrix is a meromorphic function of order two. This is in sharp contrast to the case of the congruence subgroups, in which the determinant of the scattering matrix is a meromorphic function of order one. See [10], [11], and [5].

Finally, we should mention recent important work of Wolpert ([15]) on the existence of cusp forms.

Received 13 July 1992. Revision received 26 August 1992.