Vol. 69, No. 2 DUKE MATHEMATICAL JOURNAL © February 1993

ON THE EXISTENCE OF POSITIVE SOLUTIONS OF NONLINEAR ELLIPTIC EQUATIONS—A PROBABILISTIC POTENTIAL THEORY APPROACH

Z. ZHAO

1. Introduction. In this paper we shall study the existence of solutions to the problem

$$\text{(I)} \qquad \begin{cases} \Delta u + K(x)f(u) = 0 & \text{in } D \quad (1) \\ u > 0 & \text{in } D \quad (2) \\ u = 0 & \text{on } \partial D, \quad (3) \end{cases}$$

where D is an unbounded domain in R^d $(d \geqslant 3)$. Our goal is to prove an existence theorem for problem (I) in a general setting by using Brownian path integration method and the potential theory. The following is the main result.

THEOREM 1. *Let D be an unbounded domain in R^d $(d \geqslant 3)$ with a compact Lipschitz boundary. Suppose that $f(w)$ is a continuous function in $(0, b)$ for some $0 < b \leqslant \infty$ satisfying*

$$\text{(II)} \qquad \lim_{w \to 0^+} \frac{f(w)}{w} = 0,$$

and $K(x)$ is a Green-tight function in D; namely, $K(x)$ is a Borel measurable function in D satisfying that

$$\text{(III)} \qquad \text{the family } \left\{ \frac{K(\cdot)}{|\cdot - \xi|^{d-2}} \right\} \text{ is uniformly integrable}$$

over D with the parameter $\xi \in D$. Then problem (I) *has infinitely many bounded solutions. More precisely, there exists $0 < b_0 < b$, such that for each $c \in (0, b_0]$ there exists a solution u of* (I) *satisfying*

$$\text{(4)} \qquad \lim_{|x| \to \infty} u(x) = c.$$

Remark 1. Note that there is no restriction on the sign of K and f and that there are no assumptions on the continuity and boundedness for K.

Received 19 May 1992. Revision received 25 August 1992.