Vol. 68, No. 2 DUKE MATHEMATICAL JOURNAL © November 1992

# MODULAR SYMBOLS FOR $F_q(T)$

JEREMY T. TEITELBAUM

**Introduction.** Modular symbols were invented by Manin as a tool for studying the arithmetic of modular forms for $SL_2(\mathbf{Z})$. Described in detail in [Man], these modular symbols are elements of the relative homology group $H_1(X, \text{cusps})$ where $X$ is a modular curve obtained as the quotient of the complex upper half-plane by a congruence subgroup of $SL_2(\mathbf{Z})$. This group is generated by the classes of paths joining $a$ to $b$ where $a$ and $b$ belong to $\mathbf{Q} \cup \{\infty\}$. Manin supplied an explicit finite set of generators for $H_1(X, \text{cusps})$ of this type, as well as the complete set of relations among them. This makes it very simple to calculate this $H_1$ explicitly. His construction also makes the action of the Hecke operators transparent. Using this information, one can exploit the duality between modular forms and $H_1(X, \text{cusps})$ to calculate the structure of the space of modular forms on $X$.

Modular symbols have proven to be extremely useful for a variety of purposes. For example, Mazur and Swinnerton-Dyer used modular symbols to define $p$-adic $L$-functions associated to modular forms; these $L$-functions play a role in a variety of interesting "$p$-adic Birch and Swinnerton-Dyer" type conjectures such as those studied initially in [MTT] and [MT]. A generalized, "$\Lambda$-adic" modular symbol is a crucial ingredient in the proof by Stevens and Greenberg of the (weight-two) "exceptional zero conjecture" of [MTT]. Modular symbols play a fundamental role in studying congruences between Eisenstein series and cusp forms, as in the work of Mazur ([Maz3]) and Stevens ([St]). In addition, Cremona ([Cr]) has used modular symbols as the basic tool in large scale computations of the Hecke module structure of the space of (weight-two) cusp forms for $\Gamma_0(N)$.

In this paper, we describe a theory of modular symbols for Drinfeld's "upper half-plane" $\Omega$ over $\mathbf{F}_q[[1/T]]$. Our modular symbols, which are constructed using the tree $\mathscr{T}$ of $PGL_2(\mathbf{F}_q((1/T)))$, are related to a certain class of cusp forms for $GL_2$ over (the adeles of) $\mathbf{F}_q(T)$ in the same way that Manin's symbols are related to cusp forms of weight two for $GL_2$ over the adeles of $\mathbf{Q}$. Many of the applications of Manin's modular symbols transfer immediately to our situation. For example, modular symbols yield a practical method for calculating the Hecke-module structure of certain spaces of cusp forms for $GL_2$ over $\mathbf{F}_q(T)$, making possible the construction of tables of elliptic curves over $\mathbf{F}_q(T)$ like the famous Antwerp IV tables of elliptic curves over $\mathbf{Q}$ ([MF4]). Gekeler ([Gek2]), using other methods, has constructed tables of this type, but we believe that computations based on modular symbols are more efficient for large-scale, machine-based numerical investigations.

Received 27 August 1991. Revision received 16 March 1992.
This research was partially supported by NSF grant DMS-9015523.