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MODIFIED WAVE OPERATORS AND STARK HAMILTONIANS

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1. Introduction. Quantum mechanical scattering is considered in the presence of a constant electric field. The electric field is in the $\mathbf{e}_1 = (1, 0, \ldots, 0)$ direction of n-dimensional space $\mathbb{R}^n$. The corresponding Hamiltonian (after normalization) is $H_0 = -(1/2)\Delta - x_1$ (with $\Delta = \sum_{j=1}^{n} \partial^2/\partial x_j^2$). A second Hamiltonian $H = H_0 + V$ is regarded as a perturbation of H_0 by a potential V; both Hamiltonians act as operators on $L^2(\mathbb{R}^n)$ and are self-adjoint on the appropriate domain under reasonable assumptions (as below) on V. (See [28].) The electric field's effect on spectral properties is referred to as the "Stark effect", and these operators are "Stark-effect Hamiltonians." Dollard's [5] *modified* wave operators W_D^- and W_D^+ are defined by

$$W_D^{\pm} = s\text{-}\lim_{t \to \mp\infty} e^{itH} e^{-itH_0} e^{-iX_D(t)} \tag{1.1}$$

where "s-lim" indicates that the limit is taken in the strong operator topology. (The sign reversal in (1.1) is for historical reasons [29, p. 17].) The modified wave operators are a generalization of the *Møller* wave operators,

$$W^{\pm} = s\text{-}\lim_{t \to \mp\infty} e^{itH} e^{-itH_0}, \tag{1.2}$$

which were historically used to study scattering by "short-range" potentials. The modified wave operators were introduced by J. D. Dollard [5] (in the case of no electric field, i.e., $H_0 = -\Delta/2$) to study scattering by the Coulomb potential ($V(x) = C/x$; C is a real constant). The Coulomb potential is "long-range" if there is no electric field. The motivation for introducing X_D came from the corresponding classical problem, but critically X_D must be chosen so that: (1) $W_D^{\pm}$ exist: (2) $W_D^{\pm}$ are *strongly asymptotically complete*, or more briefly, *complete*, which means their ranges both coincide with the *subspace* $L^2(\mathbb{R}^n)_c$ *of continuity* of H which is the orthogonal complement of all the eigenvectors of H; and (3) $W_D^{\pm}$ both *intertwine* H and H_0:

$$e^{-itH} W_D^{\pm} = W_D^{\pm} e^{-itH_0}.$$

Dollard further applied these modified wave operators to compare the asymptotic behavior of any state $e^{-itH}u$ ($u \in L^2(\mathbb{R}^n)_c$) to that of a corresponding free state $e^{-itH_0}v$

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